Efficient Electromagnetic Modeling of Interconnects and Packages in the dc-to-Skin Effect Limit

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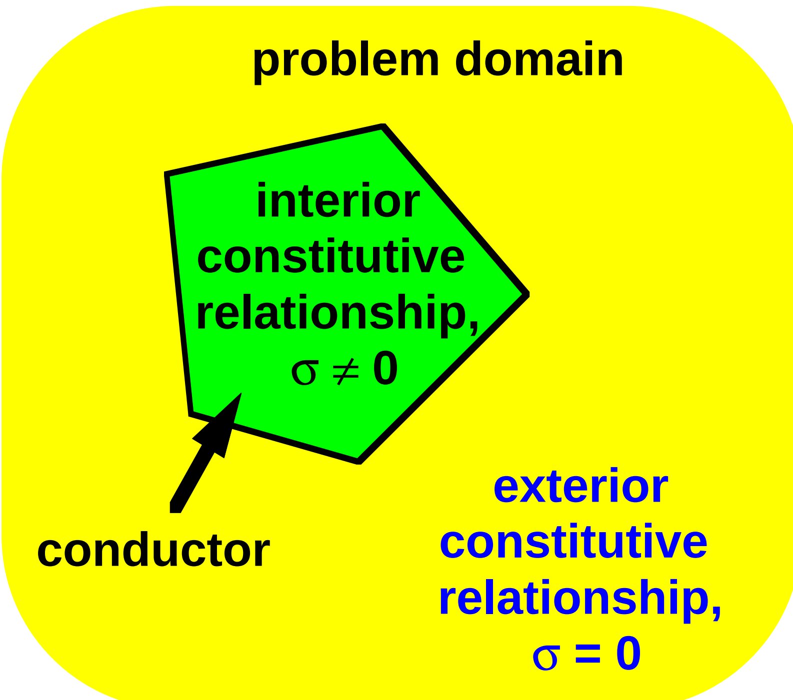
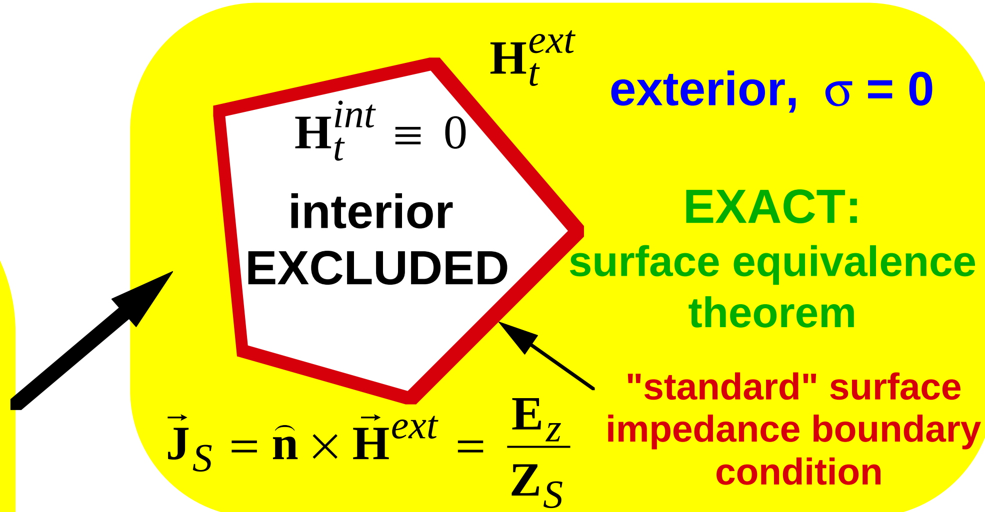
"Efficient" modeling: Elimination of conductors from solution domain via surface impedance





- definition of surface impedance: "standard" surface impedance boundary condition (SIBC)
  - surface current such that interior fields are exactly zero

Utility of surface impedance BCs

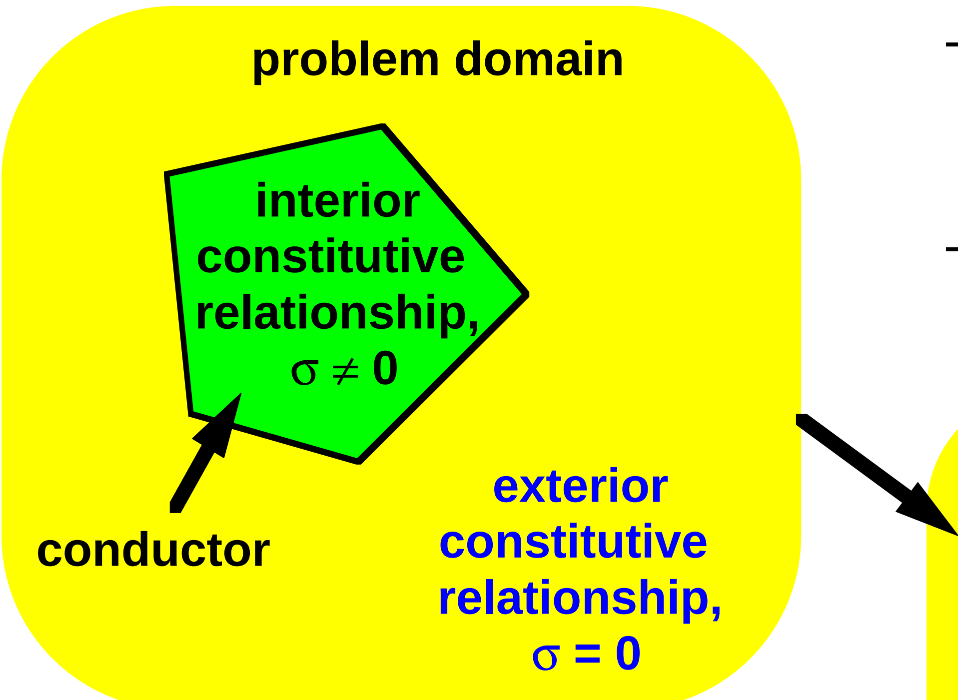
- simplifies problem only if known *a priori*
  - how well can the boundary condition be approximated?
  - most common approximation for SIBC:

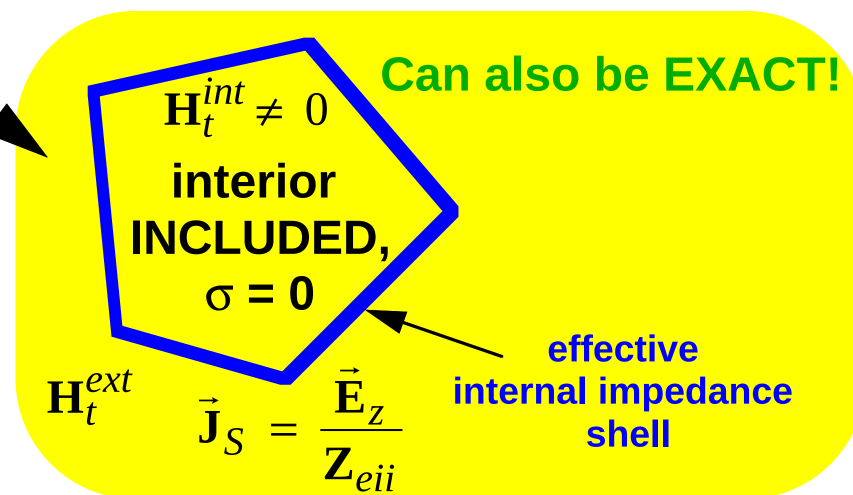
$$Z_S = \frac{(1 + j)}{\sigma \cdot \delta}$$

    - valid only at high frequency where E/H is a local property
    - requires conductor dimensions >> skin depth
  - fails at "low" frequency and when line-to-line coupling is strong

Alternative definition of conductor surface impedance: EII

- "effective" internal impedance boundary condition (EII)
  - interior of conductor replaced with non-conducting exterior medium
  - interior magnetic field is **not** identically zero





Alternative definition of conductor surface impedance: EII

- "effective" internal impedance boundary condition (EII)

$$Z_{eii}(\omega, r') = \frac{E_z(r')}{H_t^{ext}(r') - H_t^{in}(r')} = \frac{E_z(r')}{J_s(r')}$$

  - E_z and H_t^{ext} from the solution of

$$\sum_{q=1}^{m} \oint_{\Gamma_q} dr' G_{ext}^1(r,r') j\omega\mu H_t^{ext}(r') - \sum_{q=1}^{m} \oint_{\Gamma_q} dr' \left[G_{ext}^2(r,r') - 0.5\delta(r-r') \right] \cdot \left(E_z(r') + \nabla_z \Phi^q \right) = 0$$

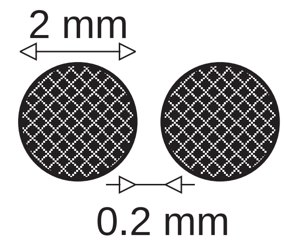
  - H_t^{in} from the solution of

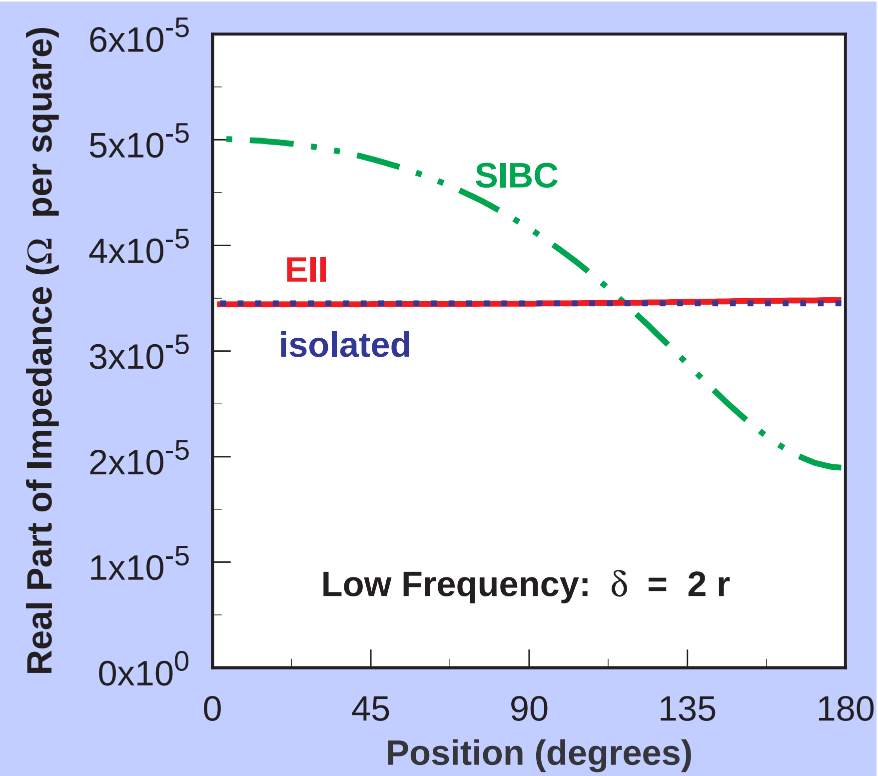
$$\oint_{\Gamma_q} dr' G_q^1(r,r') j\omega\mu H_t^{in}(r') - \oint_{\Gamma_q} dr' \left[G_q^2(r,r') + 0.5\delta(r-r') \right] E_z(r') = 0$$

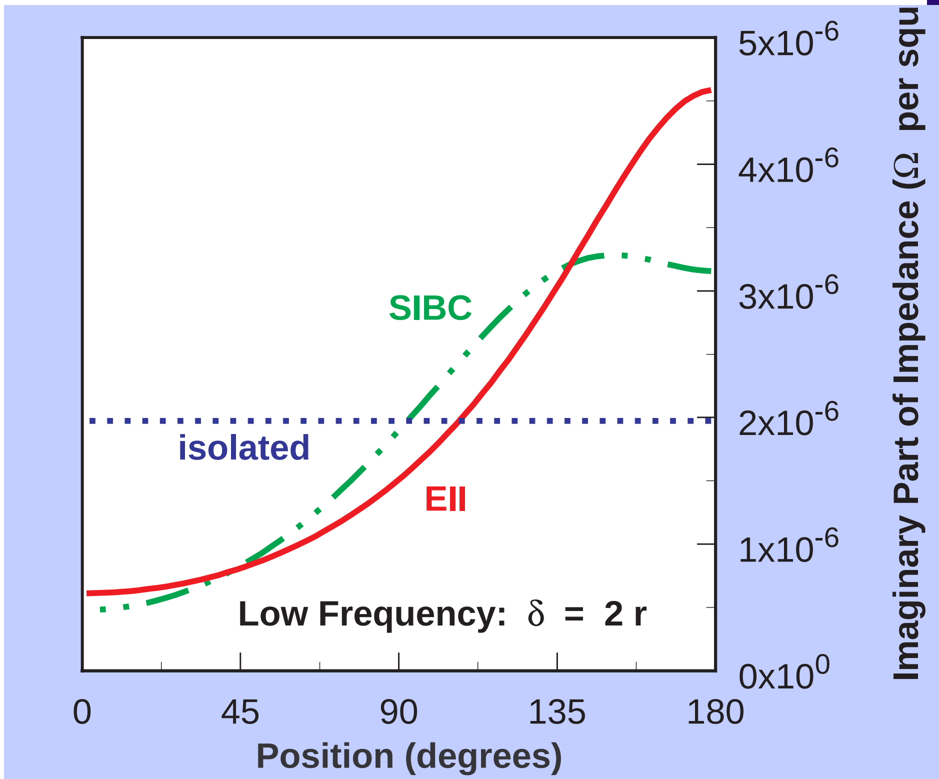
- utility of impedance boundary condition determined by how easily it can be approximated
  - isolated conductor impedance is easier to estimate than coupled line surface impedance

Surface impedance comparisons

- simple circular twin lead
- all tend to $1/\sigma\delta$ at high frequency
- EII well approximated by isolated conductor surface impedance at low frequency

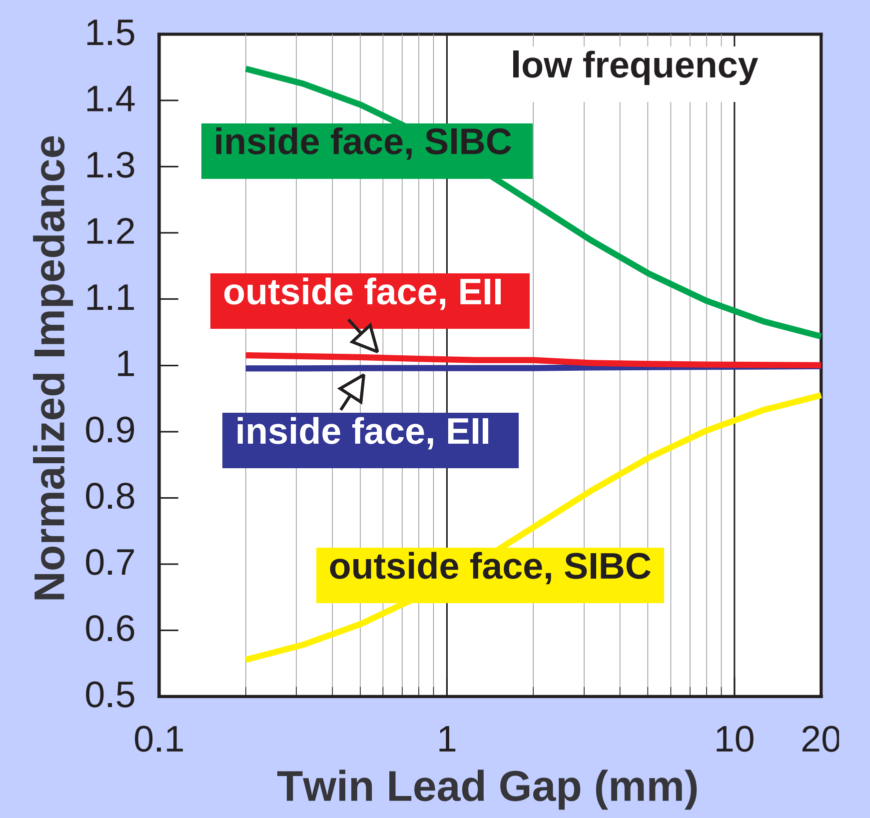


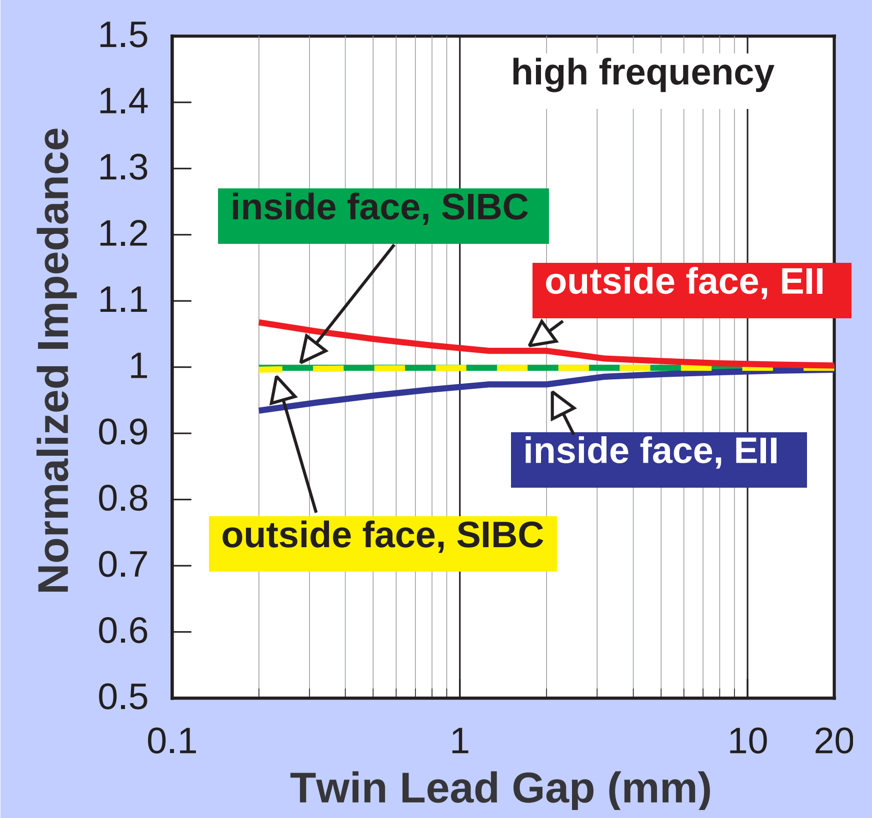




Dependence on coupling

- normalized to magnitude of isolated conductor surface impedance
- 2 mm radius circular conductors





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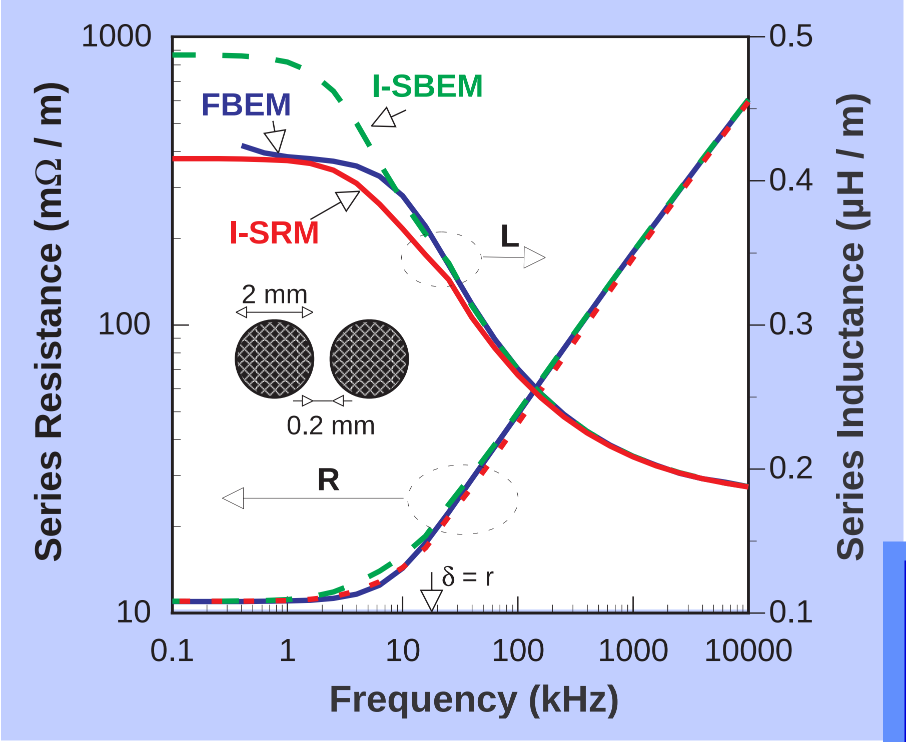
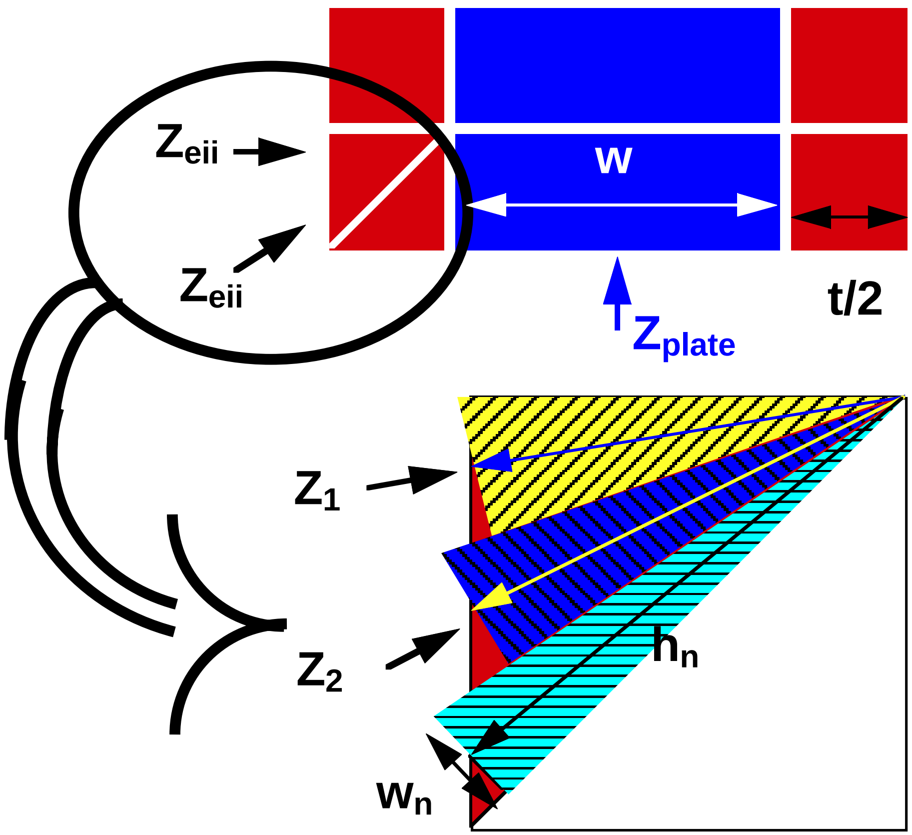
Circular twin lead comparisons



- techniques used
  - boundary element method (BEM)
  - “surface ribbon” technique (SRM)

Method	number of unkowns	CPU time (assemble)	CPU time (solve)
BEM	573	468	100
BEM with SIBC	290	18	8
SRM with EII	287	5	9

EII approximations for rectangular conductors : "Transmission line" model



- decompose bar into rectangular and square sections
- central rectangular regions: simple skin depth in "flat" plate

wide plate approximation:

$$Z_{plate} = \frac{(1+j)/(\sigma \cdot \delta)}{\mathbf{tanh}\left[(1+j)\cdot t/(2\cdot\delta)\right]} \cdot \frac{1}{w}$$

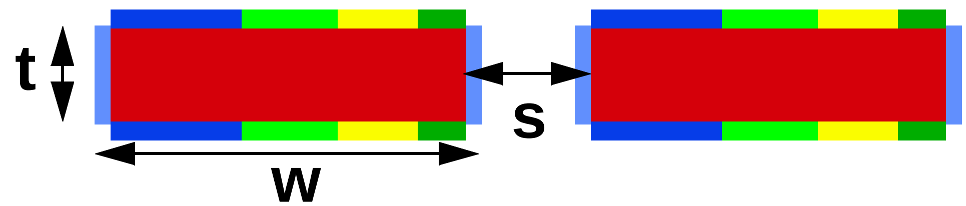
- corner regions: decompose t/2 corner into N triangles
  - exact dc resistance in limit of N large
  - high frequency behavior
    - effective thickness > t/2, current crowds toward "corner" at lower frequency

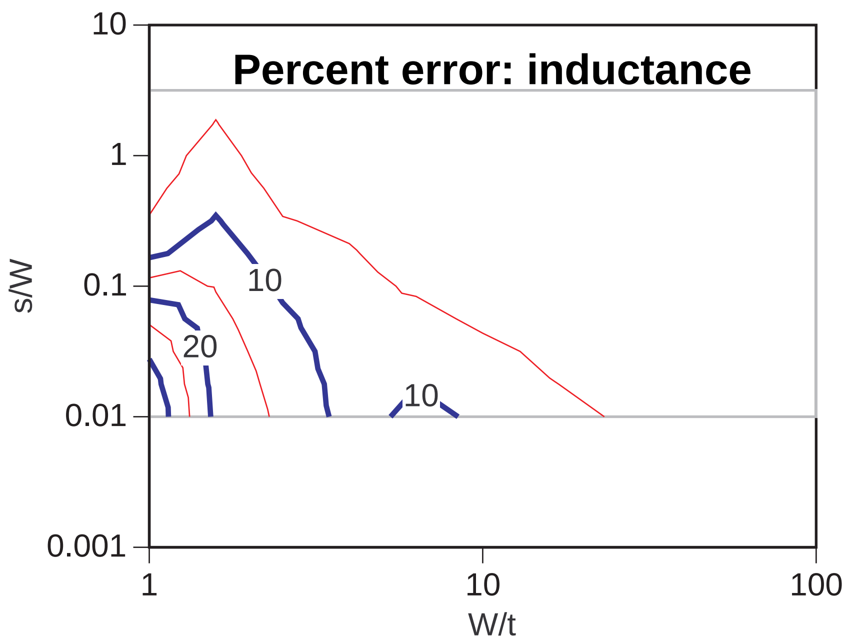
$$Z_n = \frac{j\cdot(1+j)}{\sigma\cdot\delta} \cdot \frac{\mathbf{J_0}\left(j\cdot[1+j]\cdot h_n/\delta\right)}{\mathbf{J_1}\left(j\cdot[1+j]\cdot h_n/\delta\right)} \cdot \frac{1}{w_n}$$

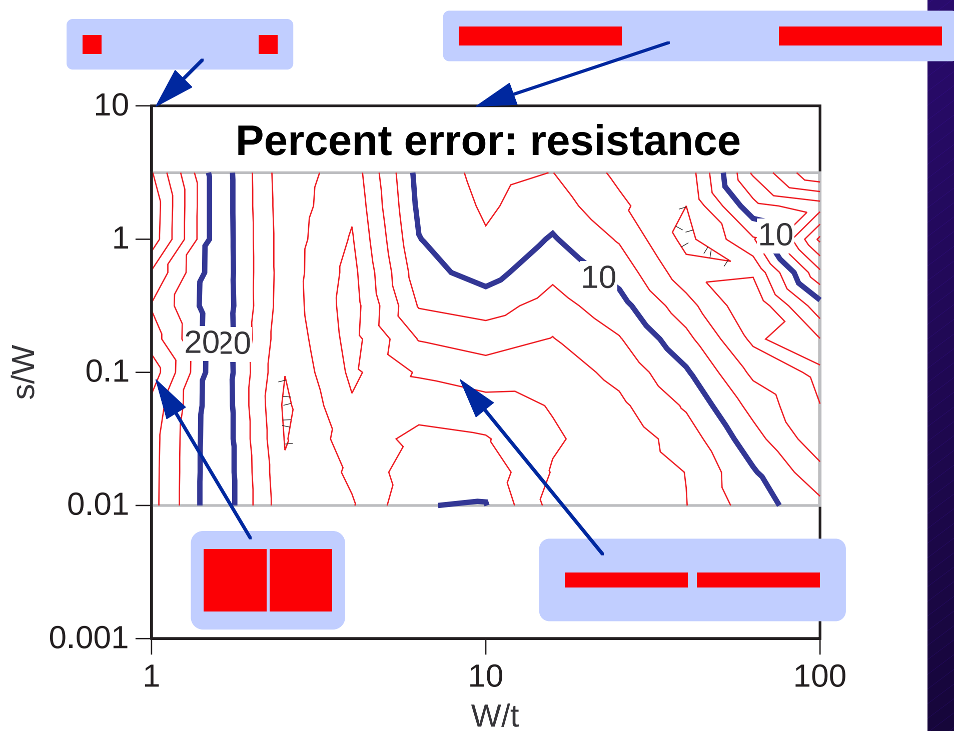
Surface ribbon method for rectangular “twin lead”

minimum ribbons:

- at most five ribbons used for “wide” faces
- one ribbon used for “narrow” faces

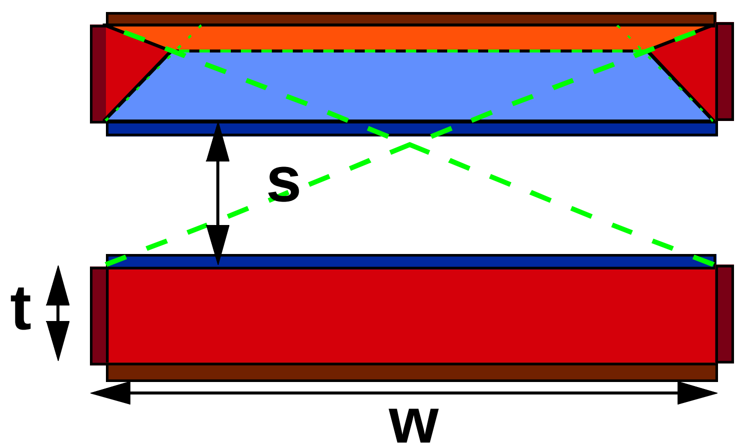


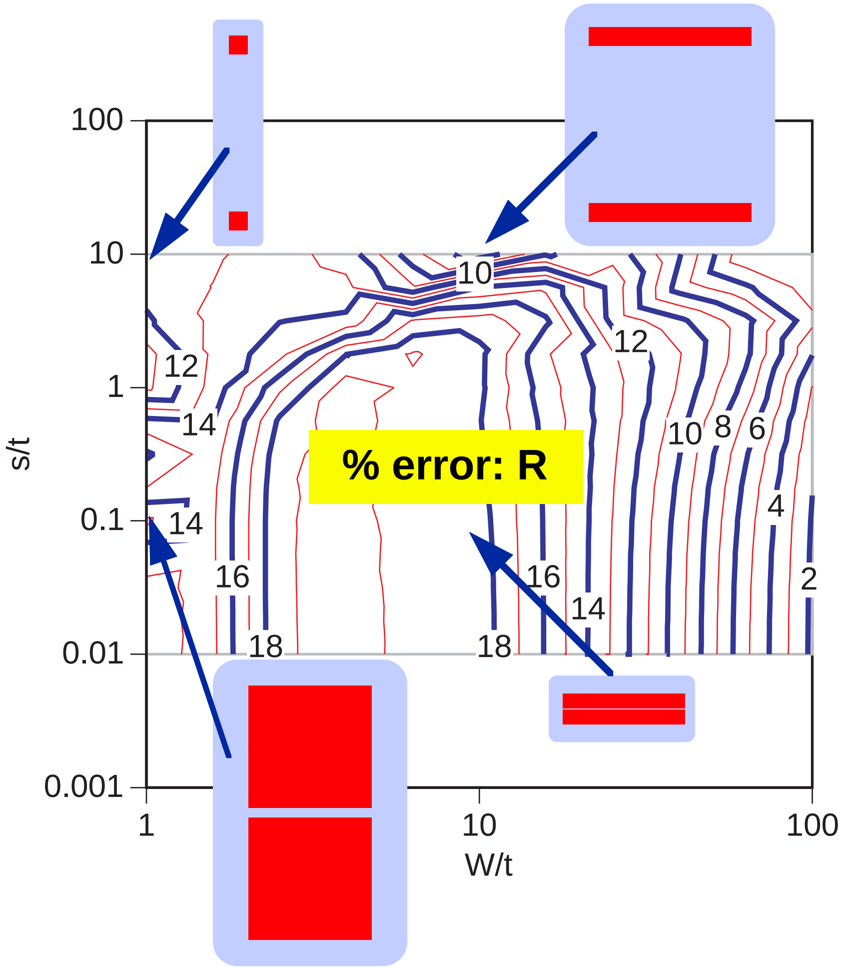




Parallel plate errors using minimum surface ribbons

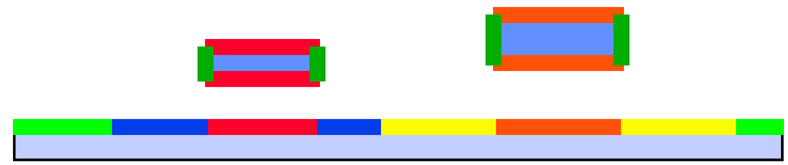
- only one ribbon per face required with appropriate EII model

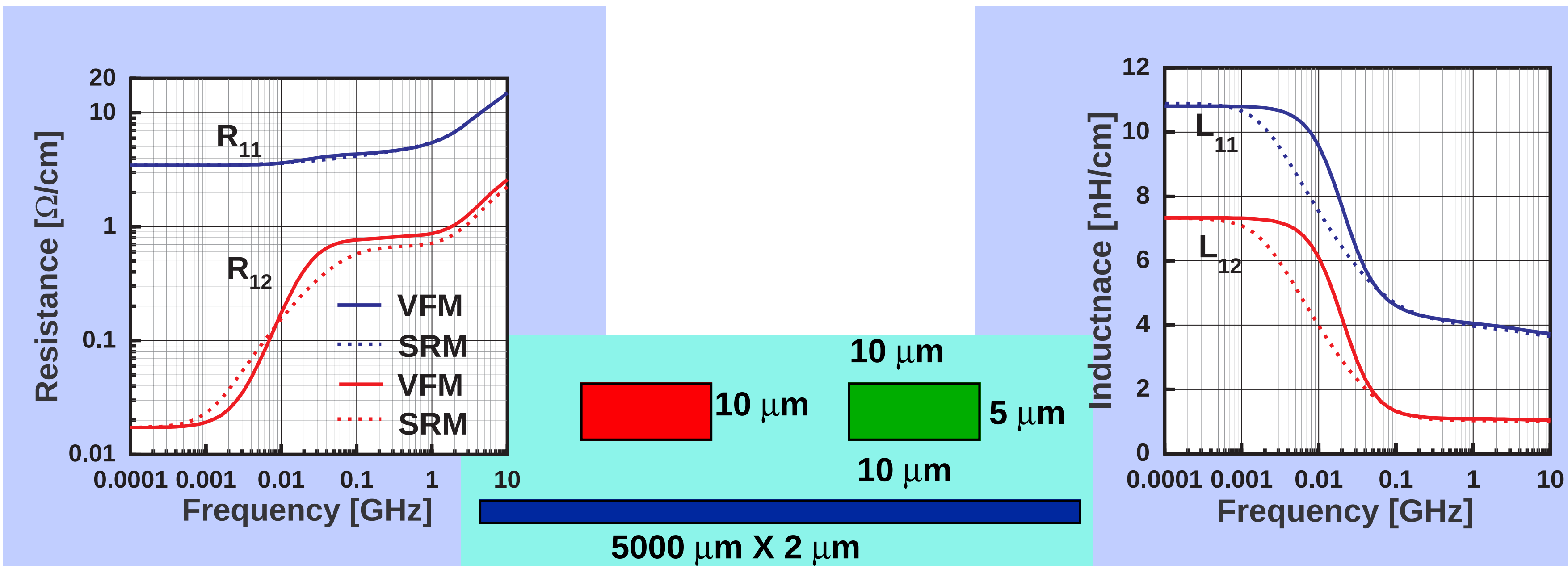




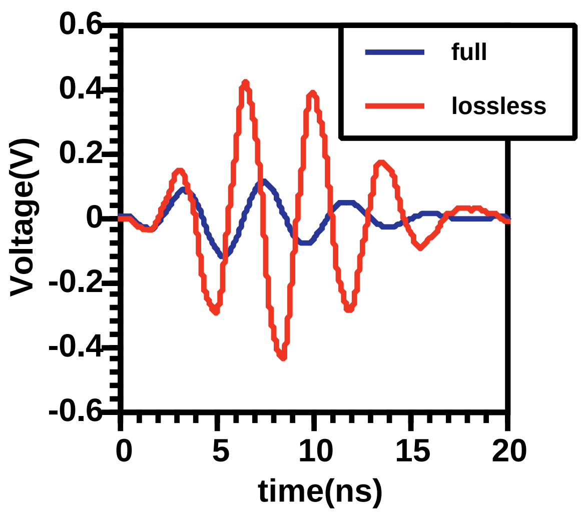
Applications: Minimum ribbon segmentation for coupled microstrip over finite ground plane

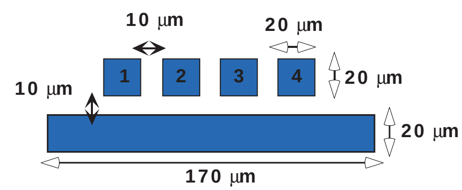
- signal lines : 4 segments each signal line (one per face)
- ground plane : 7 ~ 9 segments
- unknowns - VFM : 492 ; SRM : 15
- run time - VFM : 39 sec; SRM : 0.023 sec

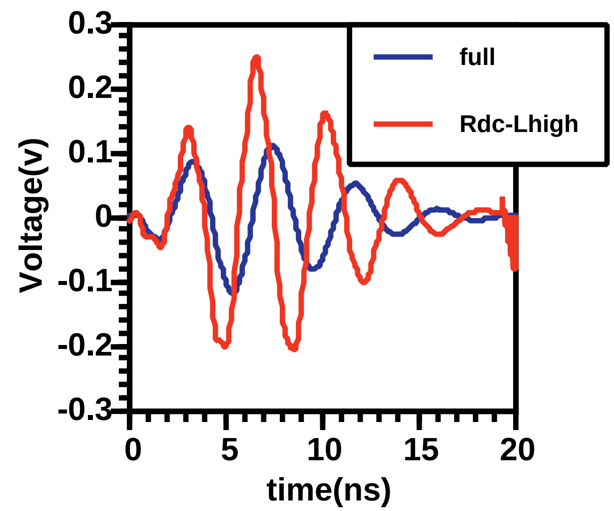




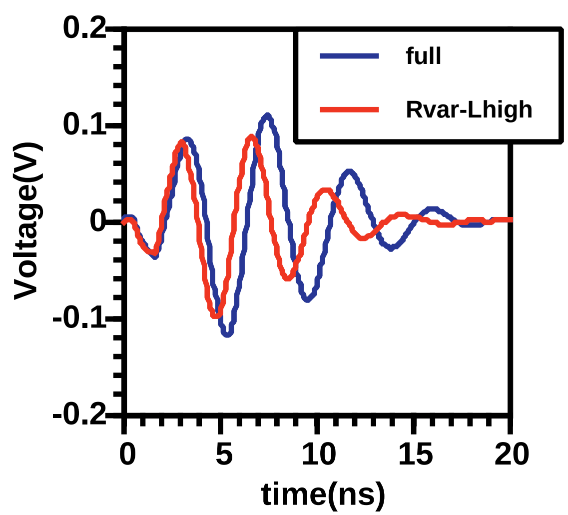
Effects of the frequency dependencies on time domain waveforms







- line 1 excited with 0.1ns rise and fall time
- measured far end of line 4 (length=0.1 m)
- R_S= 5 Ohm, C_L= 10 pF

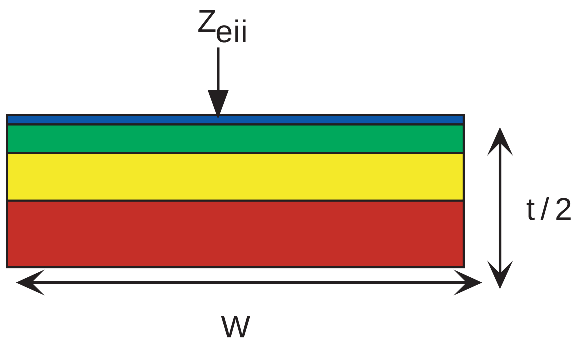


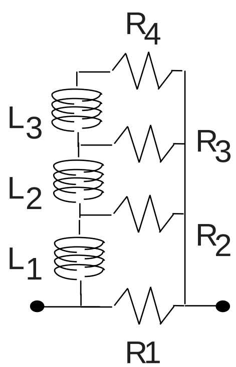
- **must include both R and L dependence on frequency / time**

Time domain simulation using surface impedance BCs

- conventional approach: convert surface impedance from frequency domain to time domain
  - can use Prony's method or Chebyshev approximation to find time domain exponential form
    - tends to produce many terms
- alternative: use compact ladder circuit to formulate time domain surface impedance boundary condition
  - use circuit to determine time domain exponential form
  - leads to time domain formulation very similar to lossless result
    - matrix containing finite conductivity effects is time independent, requires inversion only once

Equivalent circuit modeling for EII: transforming frequency domain EII into time domain





$$Z_{eii}(s) = R_1 \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

- each "isolated conductor" cross-section divided into 4 parts
- each part represented with 1 resistor and 1 inductor
- rules to determine values of circuit elements:

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = RR \qquad \frac{L_1}{L_2} = \frac{L_2}{L_3} = LL$$

  - additional constraints: correct DC resistance and inductance
  - RR and LL are empirically determined constants unique to the geometry of the conductor

Compact circuit model as a replacement for the EII

- constraints generate:

$$RR^3 + RR^2 + RR + (1 - \alpha_R) = 0$$

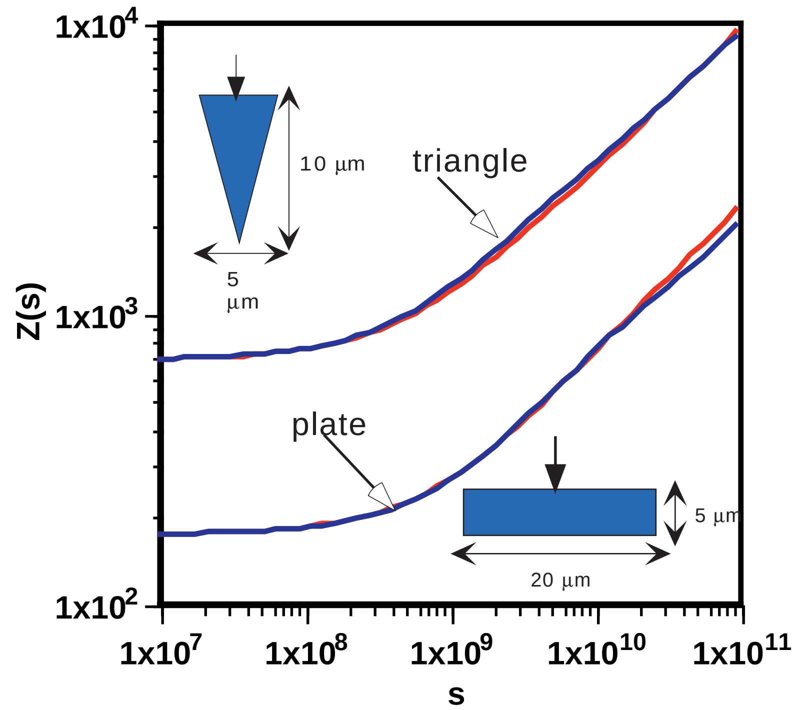
$$\left(\frac{1}{LL}\right)^2 + \left(1 + \frac{1}{RR}\right)^2 \frac{1}{LL} + \left(\left[\frac{1}{RR}\right]^2 + \frac{1}{RR} + 1\right)^2 - \alpha_L \left(\left[1 + \frac{1}{RR}\right]\left[\left\{\frac{1}{RR}\right\}^2 + 1\right]\right)^2 = 0$$

$$\alpha_R = C_1 \frac{p}{\delta_{\max}} = \frac{R_1}{R_{dc}} \qquad \delta_{\max} = \sqrt{\frac{2}{\omega_{\max}\mu_o\sigma}} \qquad \alpha_L = C_2\alpha_R = L_{dc}/L_1$$

- **p** is the conductor depth parameter, C_1 and C_2 depend on geometry:
  - triangle for corners: p = height, $C_1 = 0.56$, $C_2 = 0.315$
  - half plate for mid regions: p = thickness, $C_1 = 10.8$, $C_2 = 0.2$
  - circular conductors: p = r, $C_1 = 0.53$, $C_2 = 0.315$
- ladder values completely determined for each ribbon on conductor surface

Time domain conversion using equivalent circuit model

- equivalent circuit model
  - can be easily constructed
  - rational function in s-domain, exponential function in time domain
  - problem size can be reduced using Pade approximation: dominant pole reduction
  - time domain convolution problem can be solved using recursive properties



blue: numerical result

red: circuit model

Derivation of time domain equation

- frequency domain (s-domain) equation

$$\left[\frac{Z_{eii}}{s}\right]s[I]+[L]s[I]=-\frac{\partial}{\partial z}[V]$$

- transformation into time domain

$$L^{-1}\left(\left[\frac{Z_{eii}}{s}\right]=\left[R_1\frac{b_3s^3+b_2s^2+b_1s+b_0}{s\left(a_3s^3+a_2s^2+a_1s+a_0\right)}\right]\right)$$
$$=\left[R_1\sum K_i e^{P_i t}\right]=[\zeta(t)]$$

$$[\zeta(t)]*\frac{\partial}{\partial t}[I]+[L]\frac{\partial}{\partial t}[I]=-\frac{\partial}{\partial z}[V]$$

- time domain convolution

$$\mathrm{Y}(n\Delta t)=\mathrm{X}(n\Delta t)*Ke^{p(n\Delta t)}$$
$$=K\Delta t\cdot\mathrm{X}(n\Delta t)+e^{p\Delta t}\cdot\mathrm{Y}((n-1)\Delta t)$$

- application of recursive equation

$$[K]\frac{\partial}{\partial t}[I]+[L]\frac{\partial}{\partial t}[I]+[V_{ds}]=-\frac{\partial}{\partial z}[V]$$

$$\left[L'\right]\frac{\partial}{\partial t}[I]+\underline{\left[V_{ds}\right]}=-\frac{\partial}{\partial z}[V]$$

$$[C]\frac{\partial}{\partial t}[V]+[G]\cdot[V]=-\frac{\partial}{\partial z}[I]$$

- lossless like equation with additional voltage source
- voltage source depends on poles, residues, time step, and values from previous time step
- different simulators can be used to solve equations

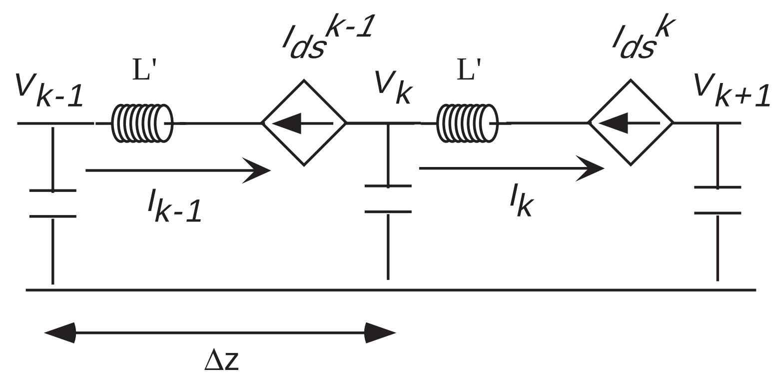
Example I: time stepping solution (FDTD)

- make finite difference approximation to the partial derivatives: extra current source compared to lossless case
- each voltage and adjacent current solution point separated by $\Delta z/2$
- Δt has to be kept small to satisfy stability condition: may not be appropriate for electrically long lines

$$[I]_k^{n+\frac{2}{3}} = [I]_k^{n+\frac{1}{2}} - \left[\frac{L'}{\Delta t}\right]^{-1}\left(\frac{[V]_{k+1}^{n+1} - [V]_k^{n+1}}{\Delta z} + [V_{ds}]_k^{n+1}\right)$$

$$= [I]_k^{n+\frac{1}{2}} - \left[\frac{L'}{\Delta t}\right]^{-1}\left(\frac{[V]_{k+1}^{n+1} - [V]_k^{n+1}}{\Delta z}\right) - \underline{[I_{ds}]}_k^{n+1}$$

$$[V]_k^{n+1} = [V]_k^n - \left[\frac{C}{\Delta t}\right]^{-1}\left(\frac{[I]_{k+1}^{n+\frac{1}{2}} - [I]_k^{n+\frac{1}{2}}}{\Delta z}\right)$$



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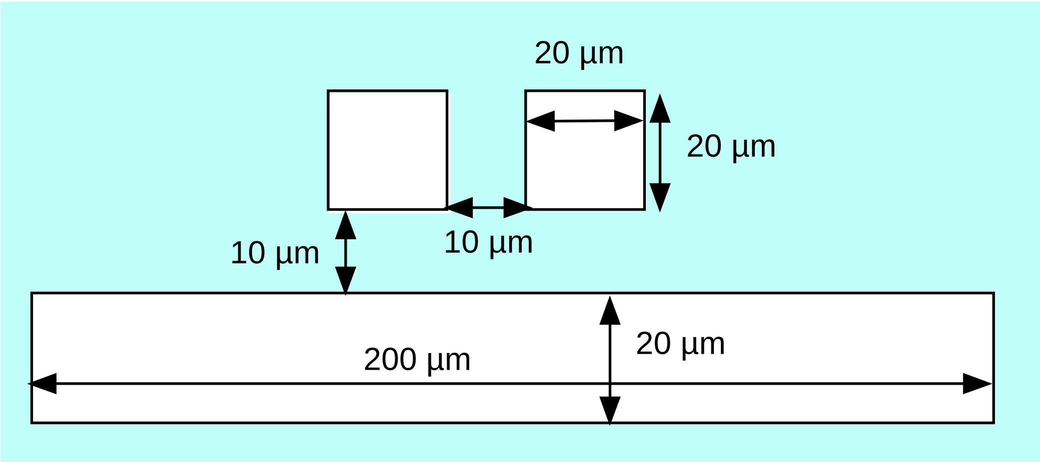
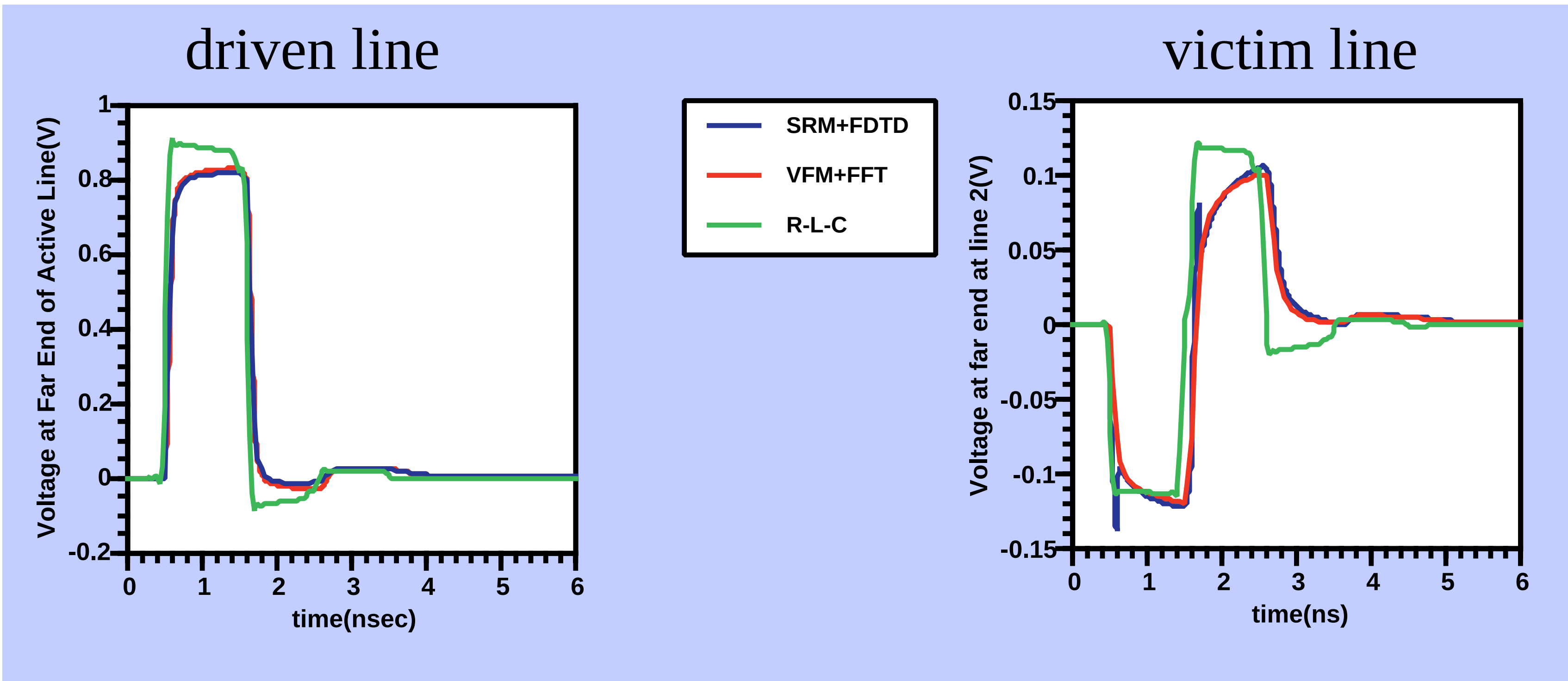
EII/Circuit approximation in FDTD

- test case geometry: two 20μm square microstrip lines, 30μm pitch, all conductivities finite



  - 10cm length, 5Ω source impedance, 50Ω load
- comparisons
  - finite difference time domain method using EII circuit derived BC
  - FFT using frequency domain dispersion curve
    - conventional current filament method (FM)
    - surface ribbon method (SRM)
  - simple RLC transmission line model (no skin effect)

Driven and victim line comparisons

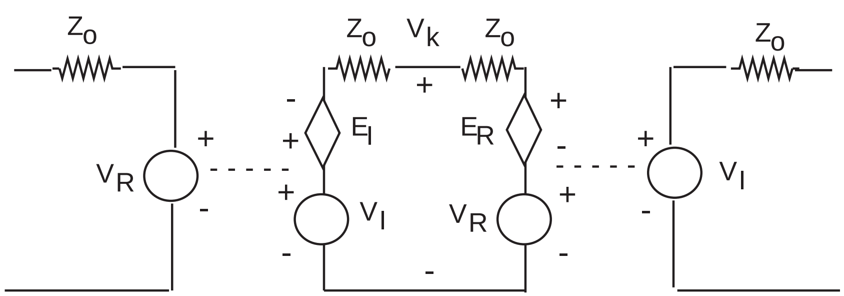


- single input pulse, 0.1nsec rise & fall times, 1nsec on time

method	pre-calculation	main calculation (FDTD or FFT)
SRM-FDTD	0.5 sec	89.5 sec
SRM-FFT	149 sec	0.5 sec
FM-FFT	163,095 sec	0.5 sec

Example II: method of characteristics (MC)

- Branin (1967) for single lossless line, Ho (1973) for multi-lossless lines, Gruodis (1979) for multi-resistive lines(R-L-C)
- objective: estimate waveform by applying Ho’s method to lossless-like equation with similar efficiency as R-L-C circuit analysis
- represent waveform as combination of incident and reflected wave
- needs additional segmentation to include effect of distributed resistance
- additional voltage source needed for skin effect model



$$[V_R]_k^n = [V]_{k+1}^{n-1} - [Z_o]\cdot[I]_{k+1}^{n-1}$$

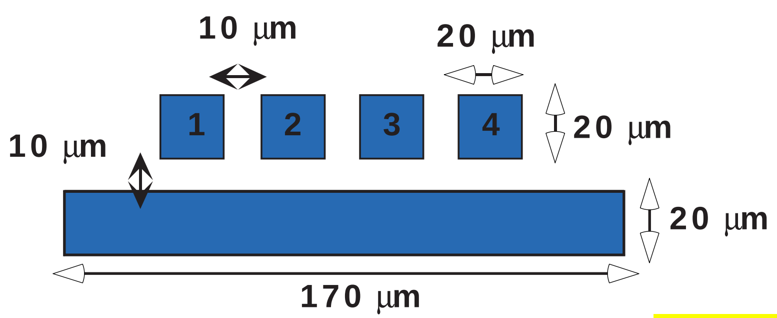
$$[V_I]_k^n = [V]_{k-1}^{n-1} + [Z_o]\cdot[I]_{k-1}^{n-1}$$

$$[Z_o] = \mathbf{v}_p \cdot [L']$$

$$[E_R]_k^n = \Delta t \cdot v_p [V_{ds}]_{k+1}^{n-1}$$

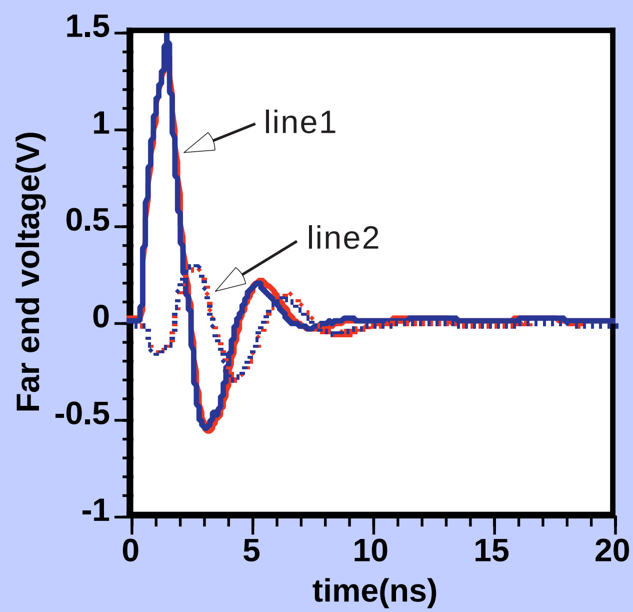
$$[E_I]_k^n = \Delta t \cdot v_p [V_{ds}]_{k-1}^{n-1}$$

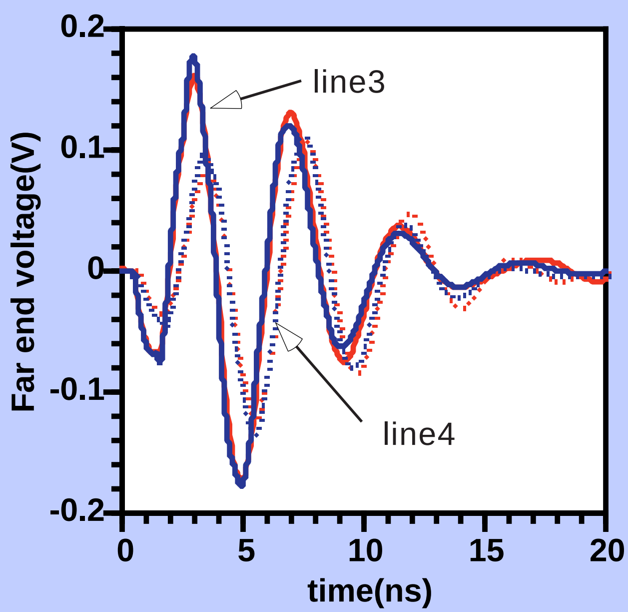
Four lines via method of characteristics



- line 1 excited, R_S=5 Ohm, C_L=10 pF
- run time comparison on Pentium II
  - SRM + MC: 7.6 sec
  - SRM + FFT: 283 sec

blue: FFT, red: MC





Efficient interconnect modeling from dc to the skin effect

- finite conductivity produces frequency dependent inductance and resistance
  - important for accurate loss or cross-talk modeling
- effective internal impedance and the surface ribbon method for dispersive R & L calculation
  - excellent approximation from dc to high frequency
  - numerically very efficient
  - applicable to n-conductor and 3-D systems
- small, frequency-independent R-L ladders can provide excellent equivalent circuit for frequency dependencies
- very efficient time domain conductor boundary condition demonstrated