EE 363M Spring 2000
Homework Set 4
Wed. March. 22
Due: Wed. March 29

1. (5 pts) Recall that for a TEM to z wave:

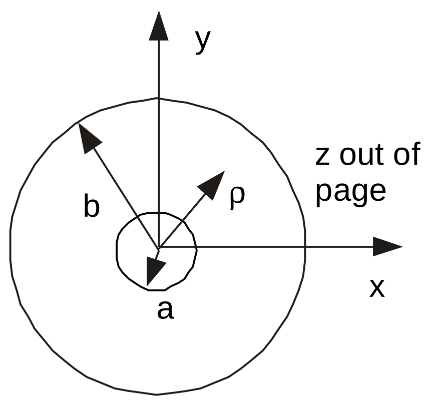
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \qquad \vec{H}_+ = \frac{1}{\eta}\left(\hat{z} \times \vec{E}_+\right) \qquad \vec{H}_- = -\frac{1}{\eta}\left(\hat{z} \times \vec{E}_-\right)$$

Assume that **E** || **x** throughout the x-y plane (i.e., that $E_y \equiv 0$ everywhere). Using the relations above show:

a) E_x is not a function of y
b) **H** as a function of **E** (write out the x and y vector components separately)
c) H_y is not a function of x
d) E_x is not a function of x

A wave TEM to z which has no field variations in the x-y plane is called a <u>uniform plane wave</u>.

2. (10 pts) A common transmission line is the coaxial cable shown in cross-section below. The radius of the inner conductor is *a*, and the cylindrical hole is of radius b. The cavity between the conductors is filled with a dielectric with $\varepsilon = \varepsilon_r\varepsilon_o$ and $\mu = \mu_o$. The outer conductor is held at ground potential (i.e. V = 0), while the inner conductor is connected to a voltage generator which applies a signal $V_o\, e^{j\omega t}$. You will find it most convenient to work in cylindrical coordinates (see summary notes below).



a) Find the electric field as a function of radial distance ρ (i.e., **E**(ρ), which will be the same as the static field in this TEM line). Use Gauss's law and show your work.

b) Verify that for this **E** field

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

c) For a TEM wave on this line, what is the **H** field? Remember for a TEM wave once you have **E** its easy to get **H**! Verify that

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0$$

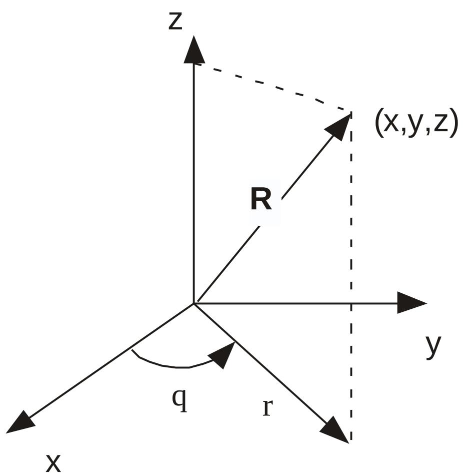
d) Find the capacitance per unit length C for this line.

e) What is the inductance per unit length L?

f) Find Z_o for this transmission line.

Note that on this line you have a TEM to z wave, that it would be considered a plane wave, but it is not a <u>uniform</u> plane wave.

Notes on cylindrical coords:



point (x,y,z) is located by

$\vec{R} = x \cdot \hat{x} + y \cdot \hat{y} + z \cdot \hat{z}$

or by

$\vec{R} = \rho \cdot \hat{\rho} + z \cdot \hat{z}$

where $\rho = (x^2 + y^2)^{1/2}$,

and $\hat{r}$ = $\cos\theta\, \hat{\mathbf{x}} + \sin\theta\, \hat{\mathbf{y}}$

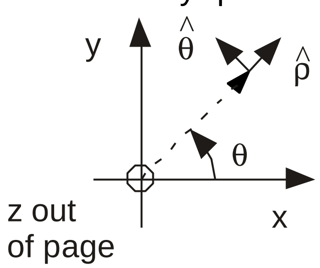
Also recall: $\cos\theta = \dfrac{x}{\sqrt{x^2+y^2}} = \dfrac{x}{\rho}$ and

$\sin\theta = \dfrac{y}{\sqrt{x^2+y^2}} = \dfrac{y}{\rho}$

In addition to the $\hat{r}$ and $\hat{\mathbf{z}}$ directions, there is an orthoganol direction $\hat{q}$, given by

$\hat{q} = -\sin\theta\, \hat{\mathbf{x}} + \cos\theta\, \hat{\mathbf{y}}$

In the x-y plane:

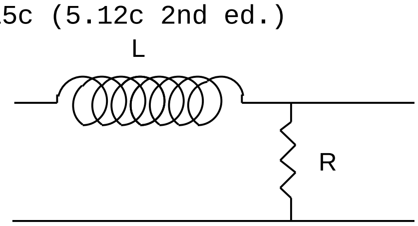


The following relations hold between the unit vectors:

$\hat{\rho} \times \hat{\theta} = \hat{z}$ $\quad \hat{\theta} \times \hat{z} = \hat{\rho}$ $\quad \hat{z} \times \hat{\rho} = \hat{\theta}$

3. (10 pts) Consider the basic T-line with series loss (i.e., $Z = R + j\omega L$, $Y = j\omega C$). For "low" frequencies (i.e., $\omega \ll R/L$), find the complex propagation constant $\gamma = \alpha + j\beta$. Sketch ω vs. β and ω vs. α diagrams (make sure to indicate over what range of ω 's your sketches are valid). Plot ω on the vertical axis, and β on the horizontal axis. Also find the phase velocity v_p and the group velocity v_g for this line, and sketch their behavior versus ω. Finally, find the "effective index of refraction" (also know as the "slow wave factor") which is defined as $n_{eff} = \beta/\beta_o$, where $\beta_o = \omega\sqrt{LC} = \omega\sqrt{\mu_o \varepsilon_o}$; sketch its behavior versus ω.

4. (5 points) Ramo, problem 5.15c (5.12c 2nd ed.)



- Find the complex propagation constant $\gamma = \alpha + j\beta$ for this line.
- Sketch ω vs. β and ω vs. α diagrams (make sure to show behavior for large β).
- Find the phase and group velocities v_p and v_g, sketch their behavior versus ω (make sure to show behavior for large ω).