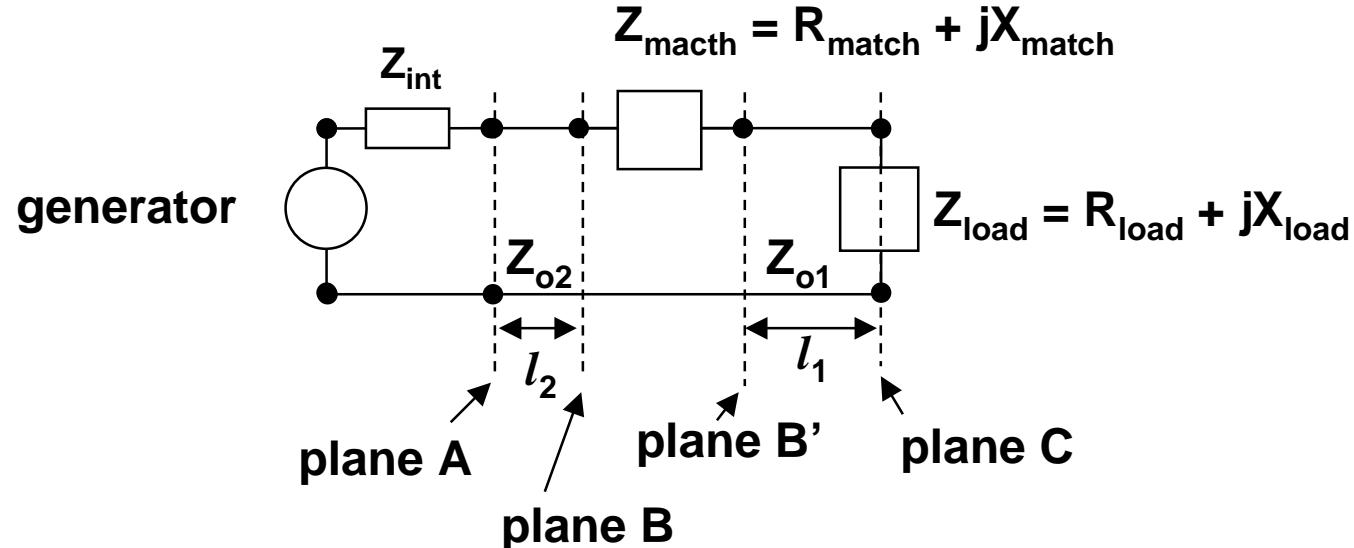


# “Equivalent” models

- can I simplify this complicated circuit?

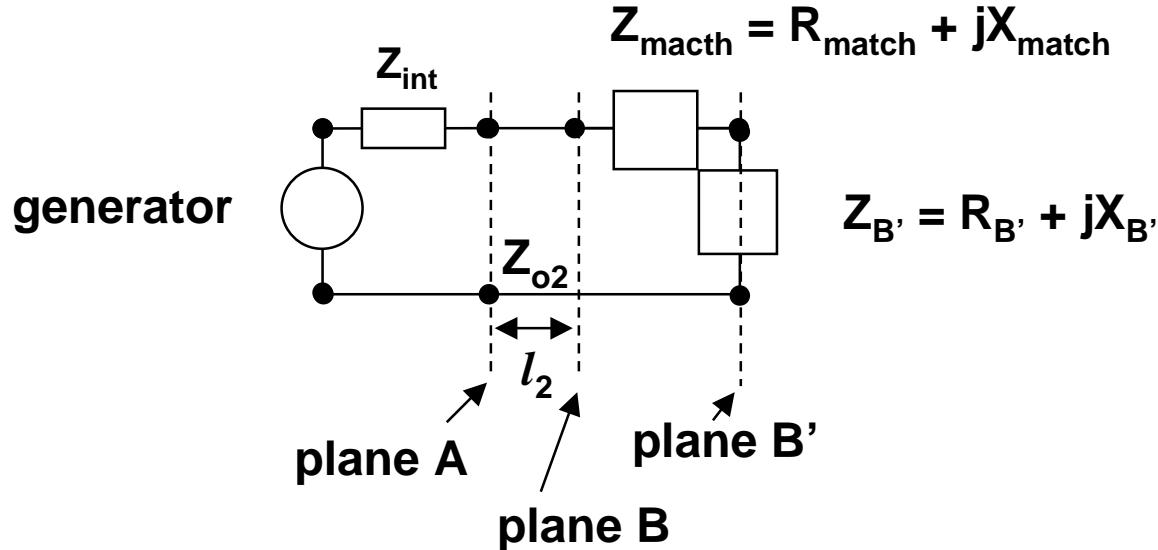


- what is  $Z (= I_{tot}/V_{tot})$  in plane C?  $Z_C = Z_{load} = R_{load} + j \cdot X_{load}$

- what about  $Z_{B'}$ ?  $Z_{B'} = Z_{o1} \cdot \frac{Z_{load} + Z_{o1} \cdot \tanh(\gamma_1 \cdot l_1)}{Z_{o1} + Z_{load} \cdot \tanh(\gamma_1 \cdot l_1)} = R_{B'} + j \cdot X_{B'}$

# “Equivalent” models

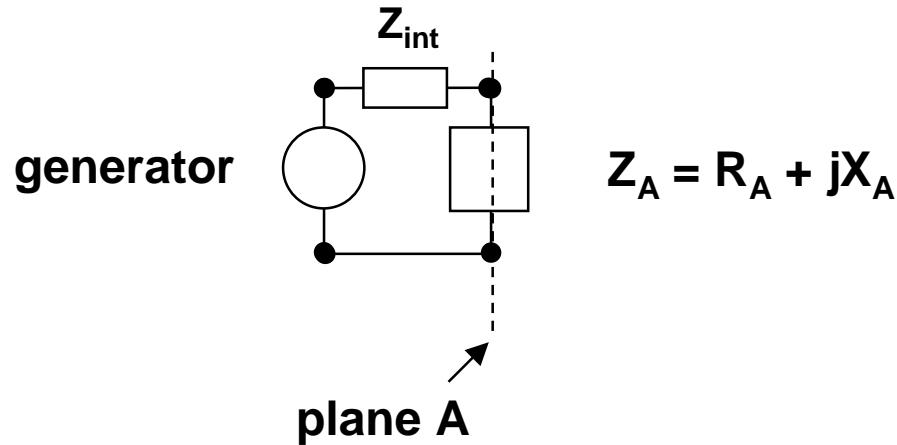
- so I can simplify the original circuit, replacing it with:



- what is  $Z$  in plane B?**  $Z_B = Z_{match} + Z_{B'} = R_{match} + R_{B'} + j \cdot (X_{match} + X_{B'})$
- what about  $Z_A$ ?**  $Z_A = Z_{o2} \cdot \frac{Z_B + Z_{o2} \cdot \tanh(\gamma_2 \cdot l_2)}{Z_{o2} + Z_B \cdot \tanh(\gamma_2 \cdot l_2)} = R_A + j \cdot X_A$

# “Equivalent” models

- so I can simplify the circuit even further:

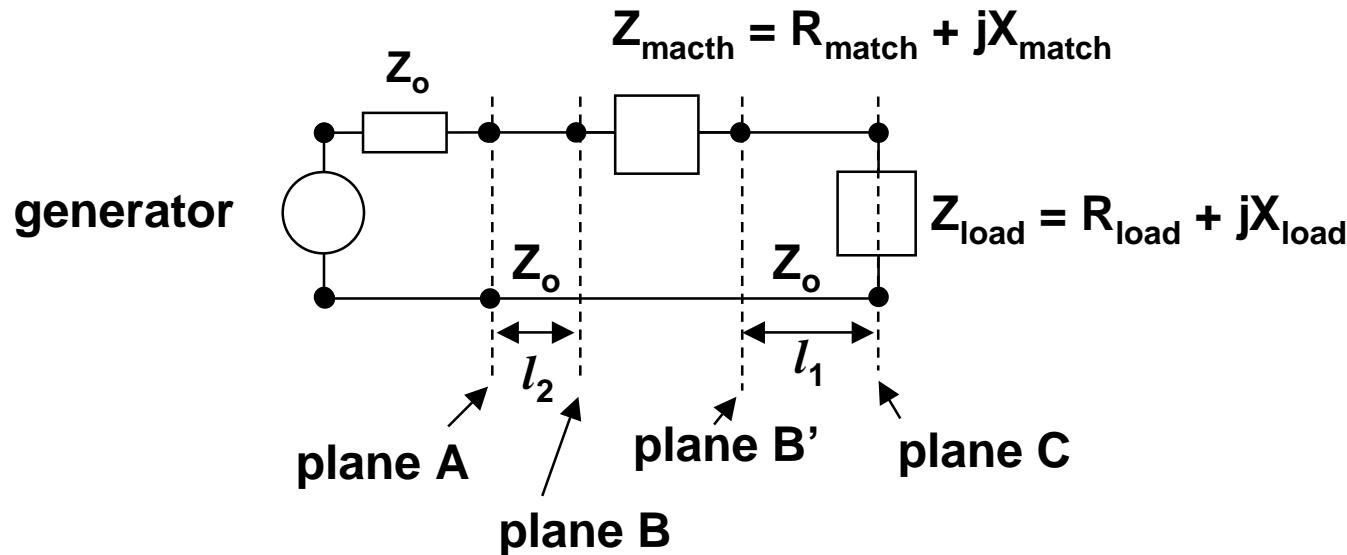


- what is the reflection back to the generator?
- then to have zero reflection we must have  $Z_A = Z_{int}$

$$\rho = \frac{Z_A - Z_{int}}{Z_A + Z_{int}}$$

# Example for series matching

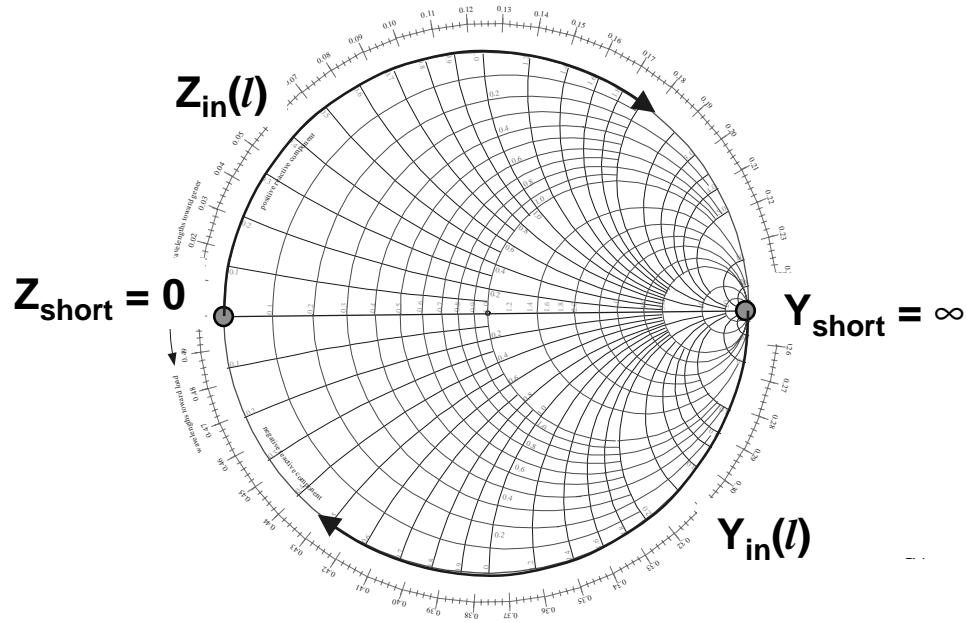
- let's assume both T-lines have the same characteristic impedance  $Z_o$ , and that the generator internal impedance is also  $Z_o$



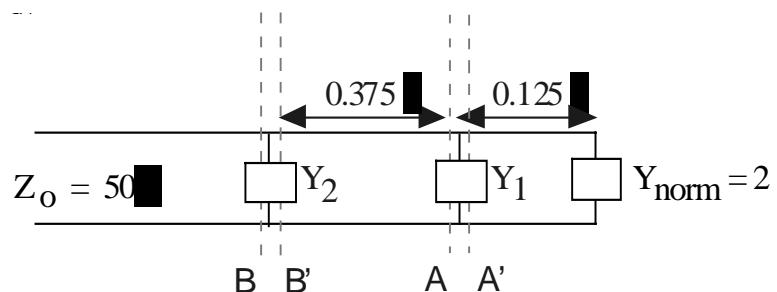
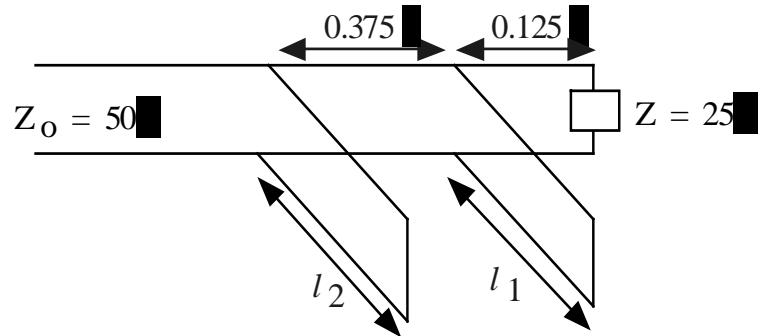
- no reflection at plane A:**  $\rho = \frac{Z_A - Z_o}{Z_A + Z_o} = 0 \rightarrow Z_A = Z_o$
- does  $l_2$  matter?**
  - NOT THIS TIME!**  $Z_A = Z_o \cdot \frac{Z_B + Z_o \cdot \tanh(\gamma_2 \cdot l_2)}{Z_o + Z_B \cdot \tanh(\gamma_2 \cdot l_2)} = Z_o \rightarrow Z_B = Z_o$

# Double stub tuner problem

- using two “stub” tuning elements, match the load
  - stubs produce purely reactive impedances (or admittances)



- work whole problem in admittance since everything is in parallel

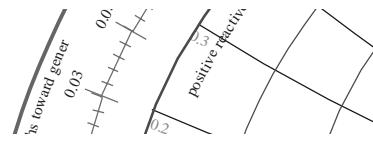


# solution

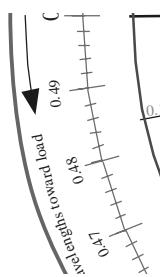
- first step: transform  $g=1$  circle

rotate all points of  $g = 1$  circle back towards load

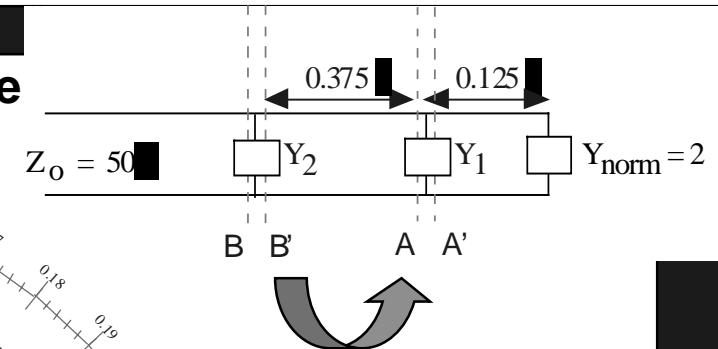
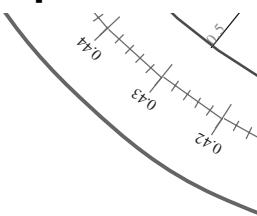
to plane A:  $3/8 \lambda$



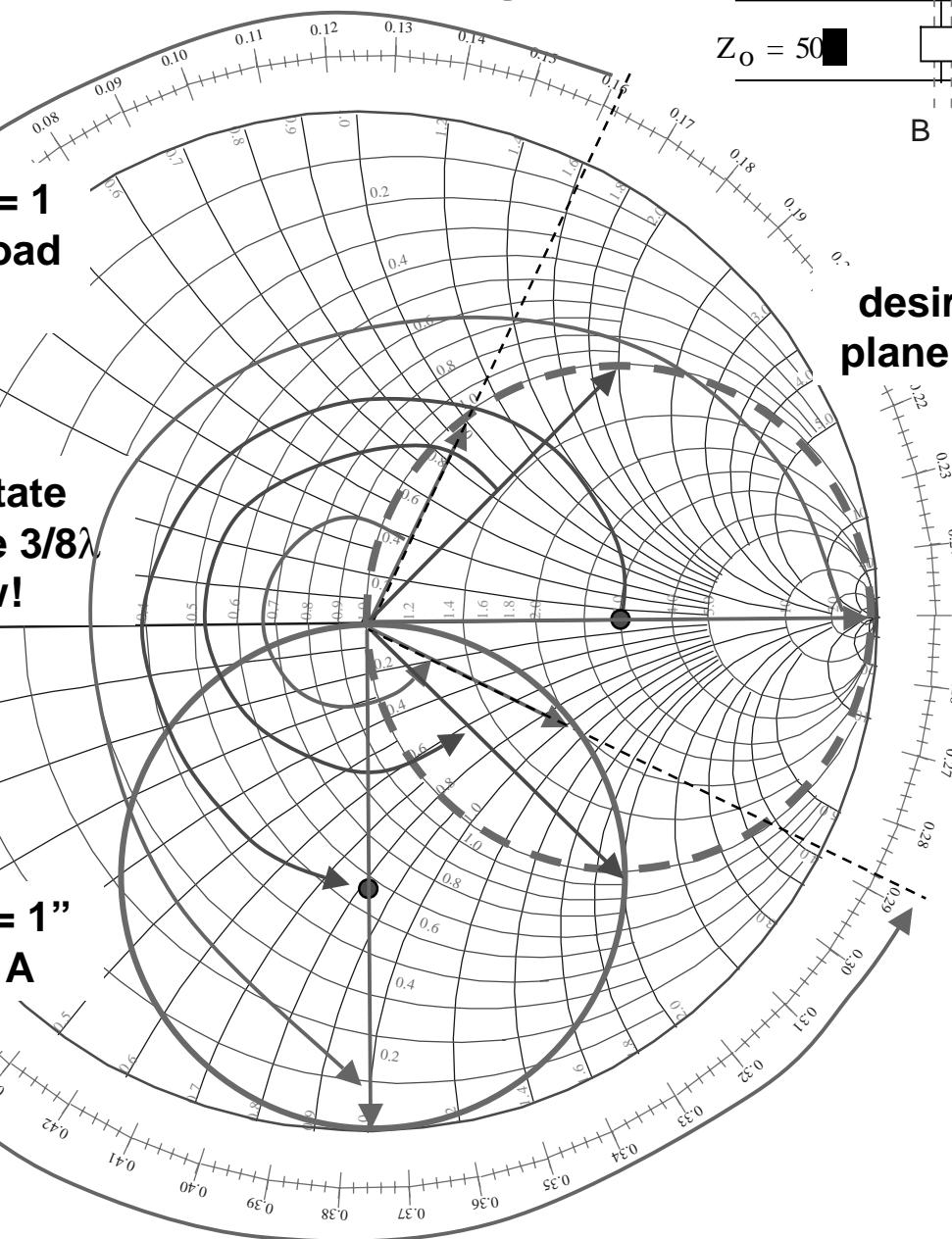
short-cut: rotate center of circle  $3/8\lambda$  and redraw!



transformed “ $g = 1$ ” circle in plane A

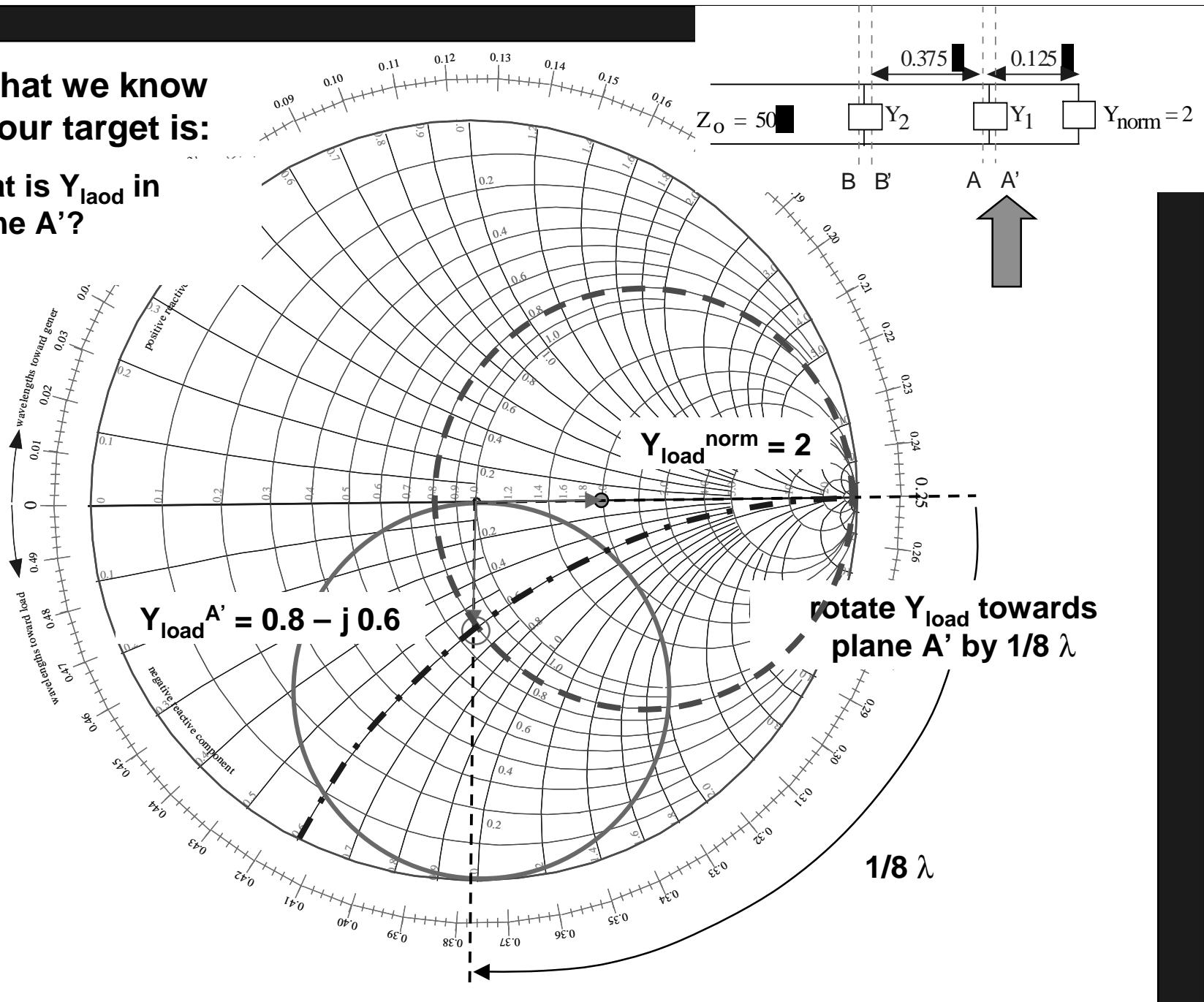


desired location in plane B':  $g = 1$  circle



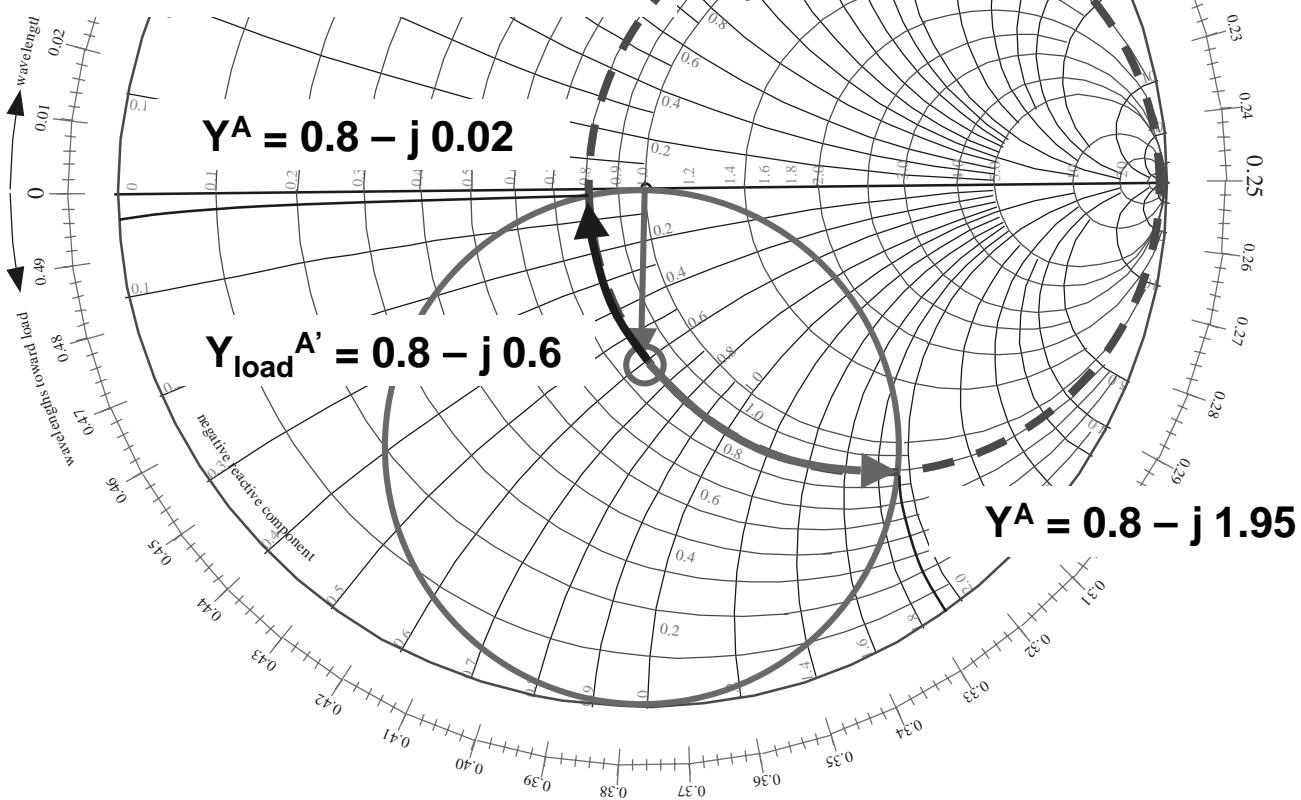
**now that we know  
what our target is:**

- **what is  $Y_{load}$  in  
plane A'?**



**we need to get from  
plane A' to A**

- what is  $Y_1$  so that in plane A we'll be on the transformed  $g=1$  circle?
  - since matching element is purely reactive, must stay on circle of constant  $g$ !



# impedance in plane A

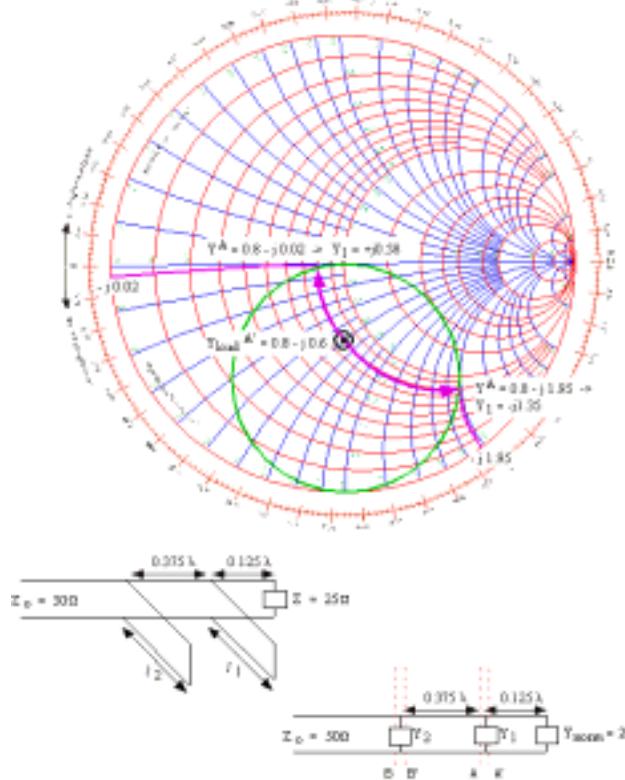
- we now have what we need to get  
 $Y_1$ :**

$$Y^A = Y_{\text{load}}^A + Y_1 \Rightarrow Y_1 = Y^A - Y_{\text{load}}^A$$

$$Y_{\text{load}}^A = 0.8 - j \cdot 0.6$$

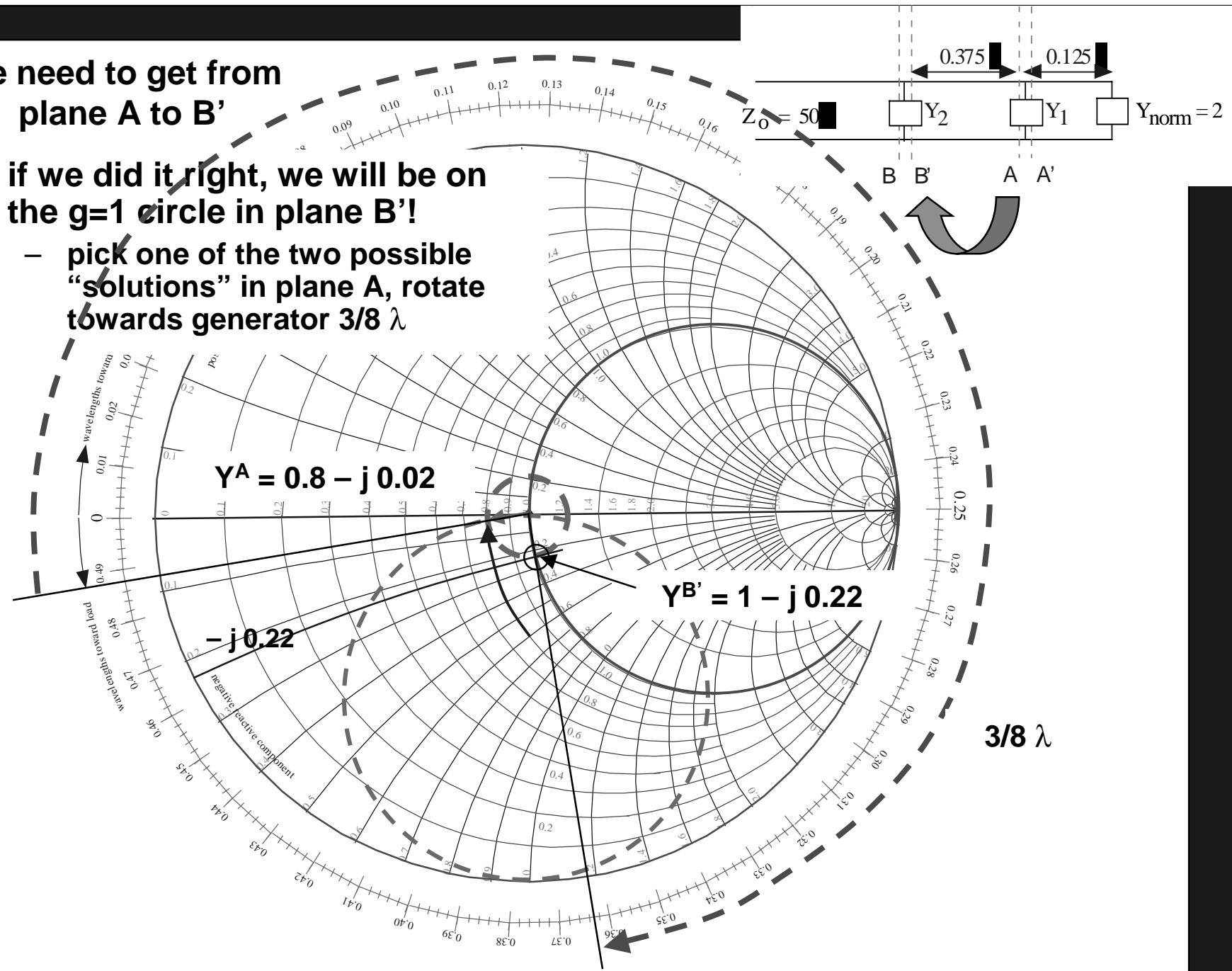
$$Y^A = \begin{cases} 0.8 - j \cdot 0.02 \\ \text{or} \\ 0.8 - j \cdot 1.95 \end{cases}$$

$$Y_1 = \begin{cases} (0.8 - j \cdot 0.02) - (0.8 - j \cdot 0.6) \\ \text{or} \\ (0.8 - j \cdot 1.95) - (0.8 - j \cdot 0.6) \end{cases} = \begin{cases} j \cdot 0.58 \\ \text{or} \\ -j \cdot 1.35 \end{cases}$$



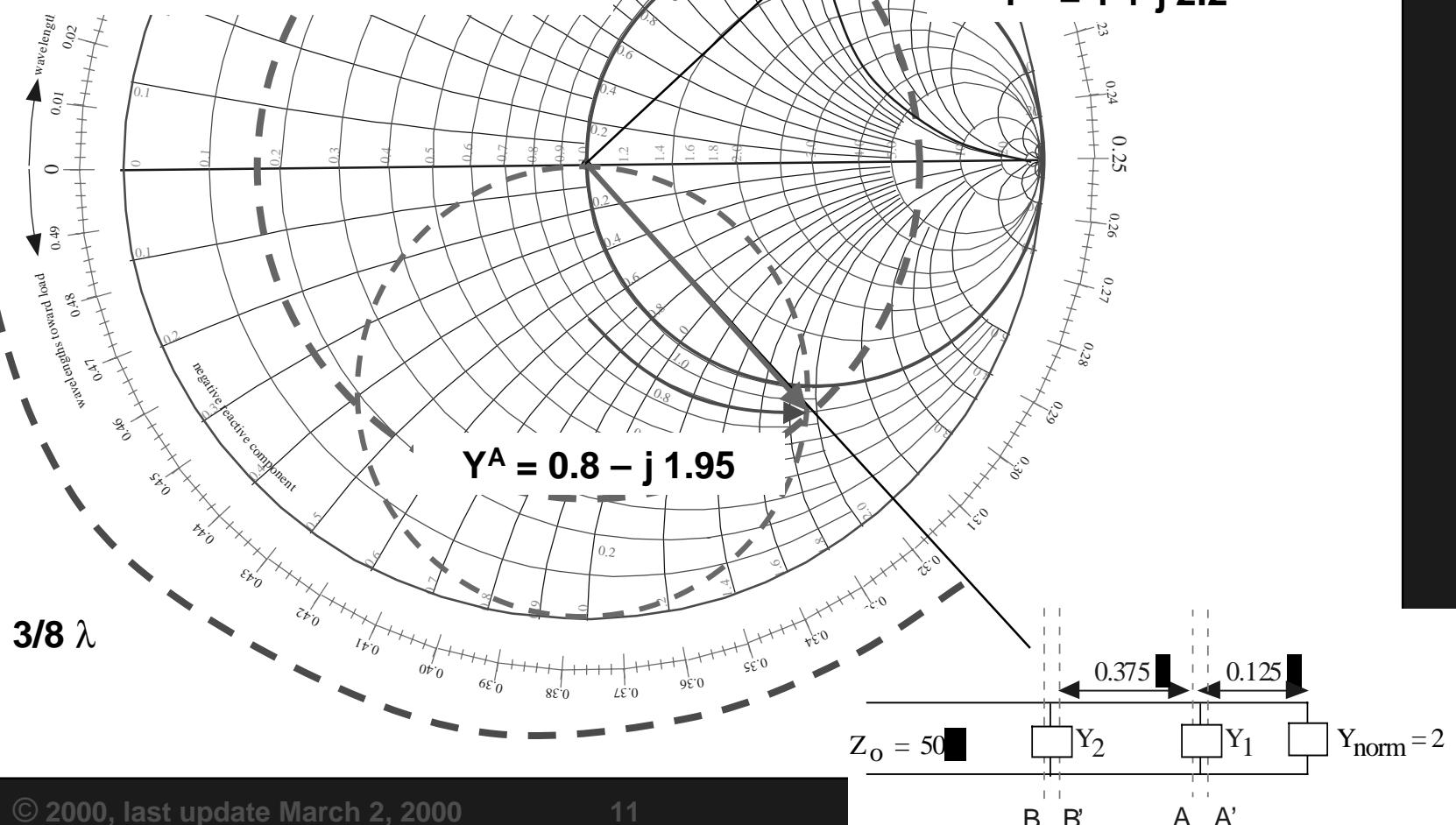
**we need to get from  
plane A to B'**

- if we did it right, we will be on  
the  $g=1$  circle in plane B'
  - pick one of the two possible  
“solutions” in plane A, rotate  
towards generator  $3/8 \lambda$

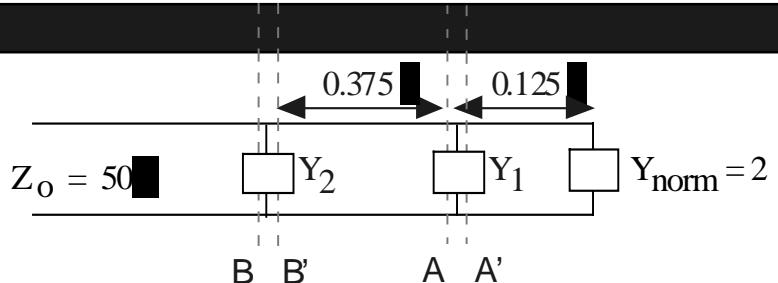


**we need to get from  
plane A to B'**

- if we did it right, we will be on  
the  $g=1$  circle in plane B'!
  - check the other “solution” in  
plane A, rotate towards  
generator  $3/8 \lambda$



## impedance in planes B' and B



$$Y_1 = \begin{cases} j \cdot 0.58 \Rightarrow Y^A = 0.8 - j \cdot 0.02 & \Rightarrow Y^{B'} = 1 - j \cdot 0.22 \\ \text{or} & \text{or} \\ -j \cdot 1.35 \Rightarrow Y^A = 0.8 - j \cdot 1.95 & \Rightarrow Y^{B'} = 1 + j \cdot 2.2 \end{cases}$$

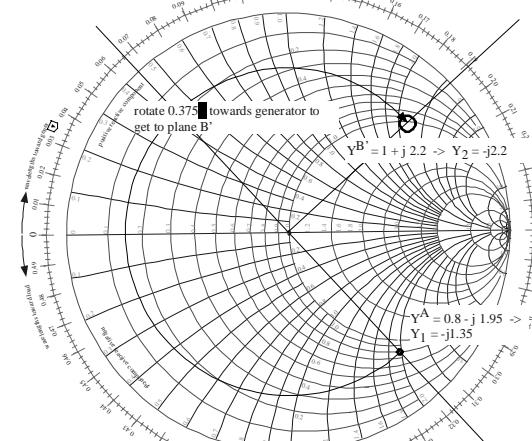
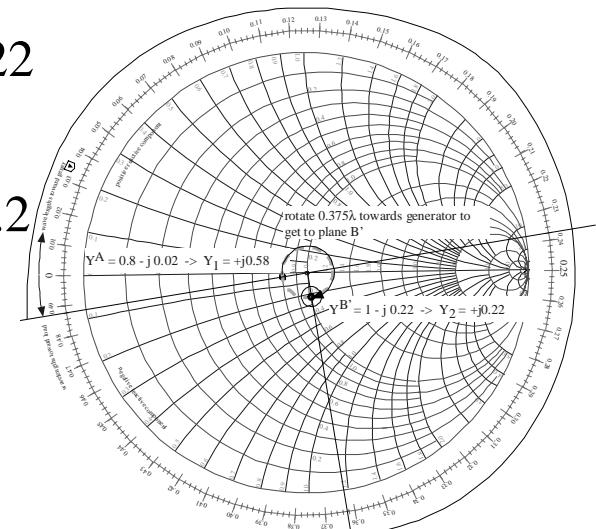
- and the admittance in plane B is just

$$Y^B = Y^{B'} + Y_2 \Rightarrow Y_2 = Y^B - Y^{B'}$$

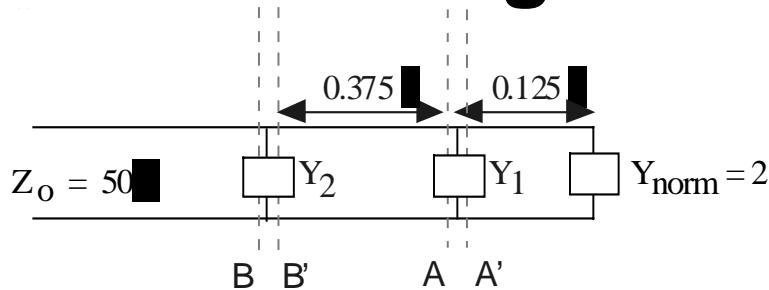
- for “no reflection” we need  $Y^B = 1$

$$Y_2 = 1 - Y^{B'}$$

$$Y_2 = \begin{cases} (1) - (1 - j \cdot 0.22) & = +j \cdot 0.22 \\ \text{or} & \text{or} \\ (1) - (1 + j \cdot 2.2) & = -j \cdot 2.2 \end{cases}$$



# Stub lengths for parallel matching



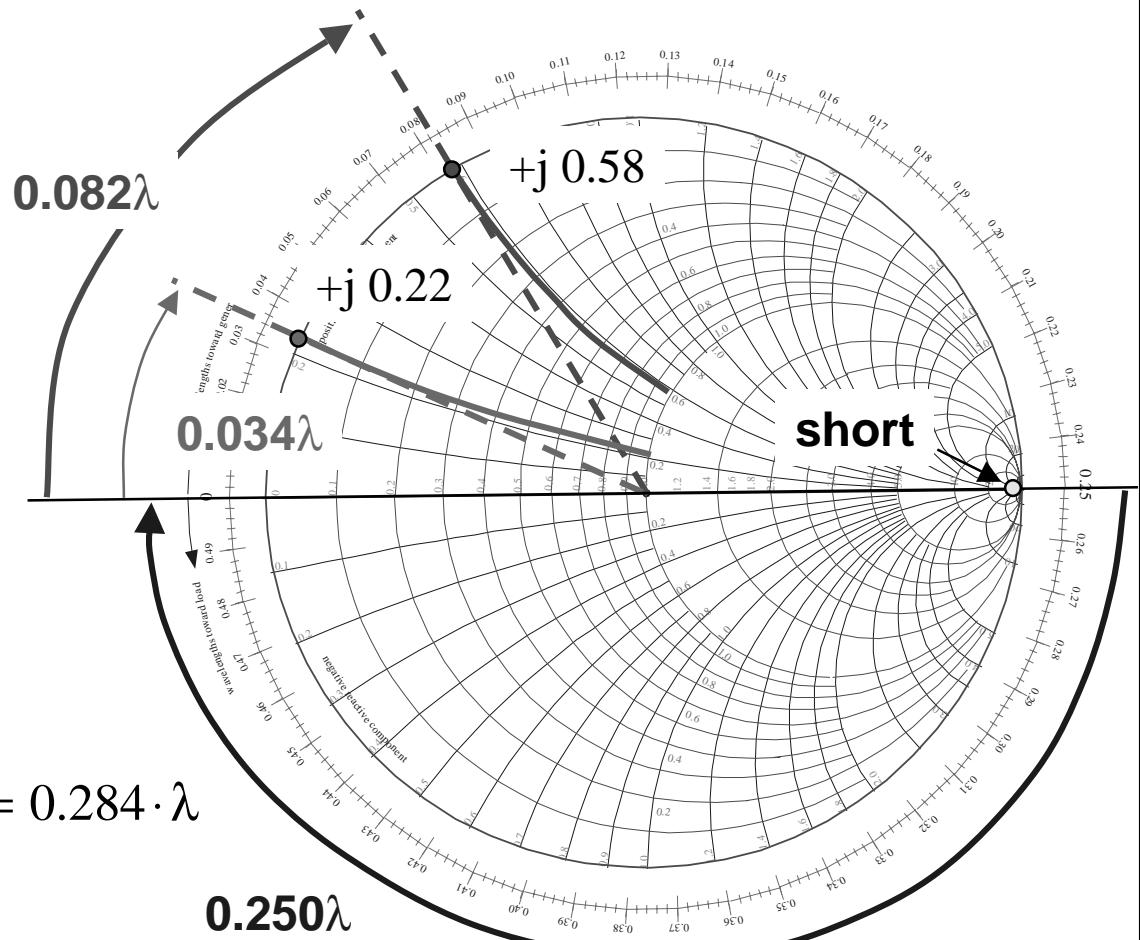
$$Y_1 = j \cdot 0.58$$

$$\Rightarrow l_1 = (0.25 + 0.082) \cdot \lambda = 0.332 \cdot \lambda$$

- plot admittance of stub
  - short: infinite admittance
- rotate towards input end of stub (clockwise) to desired susceptance

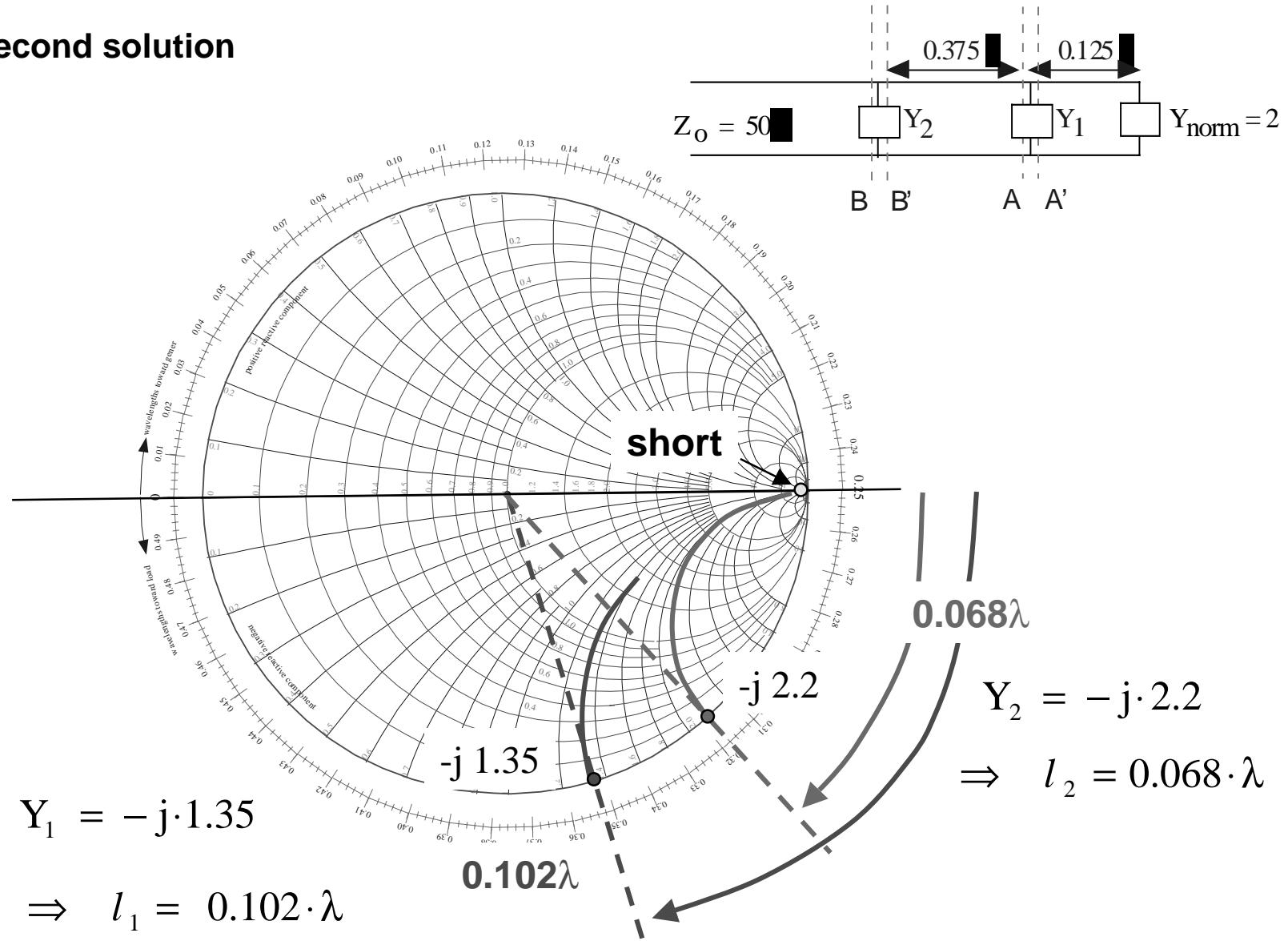
$$Y_2 = j \cdot 0.22$$

$$\Rightarrow l_2 = (0.25 + 0.034) \cdot \lambda = 0.284 \cdot \lambda$$



# Stub lengths for parallel matching

- second solution

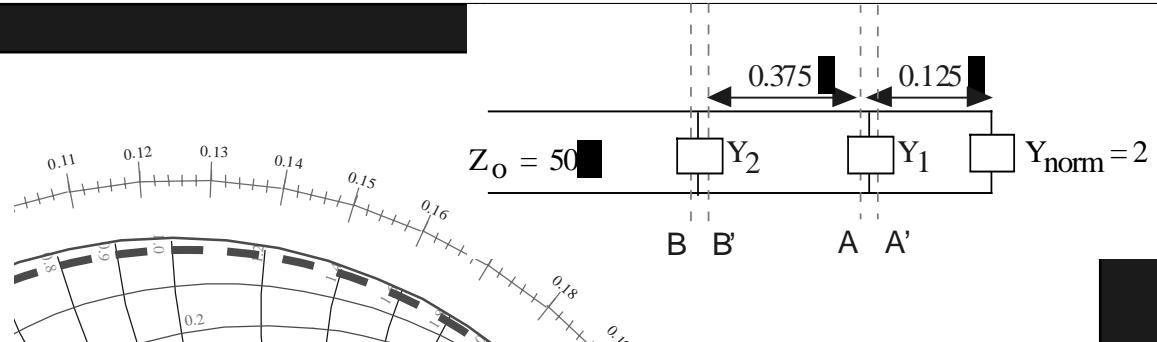


$$Y_1 = -j \cdot 1.35$$

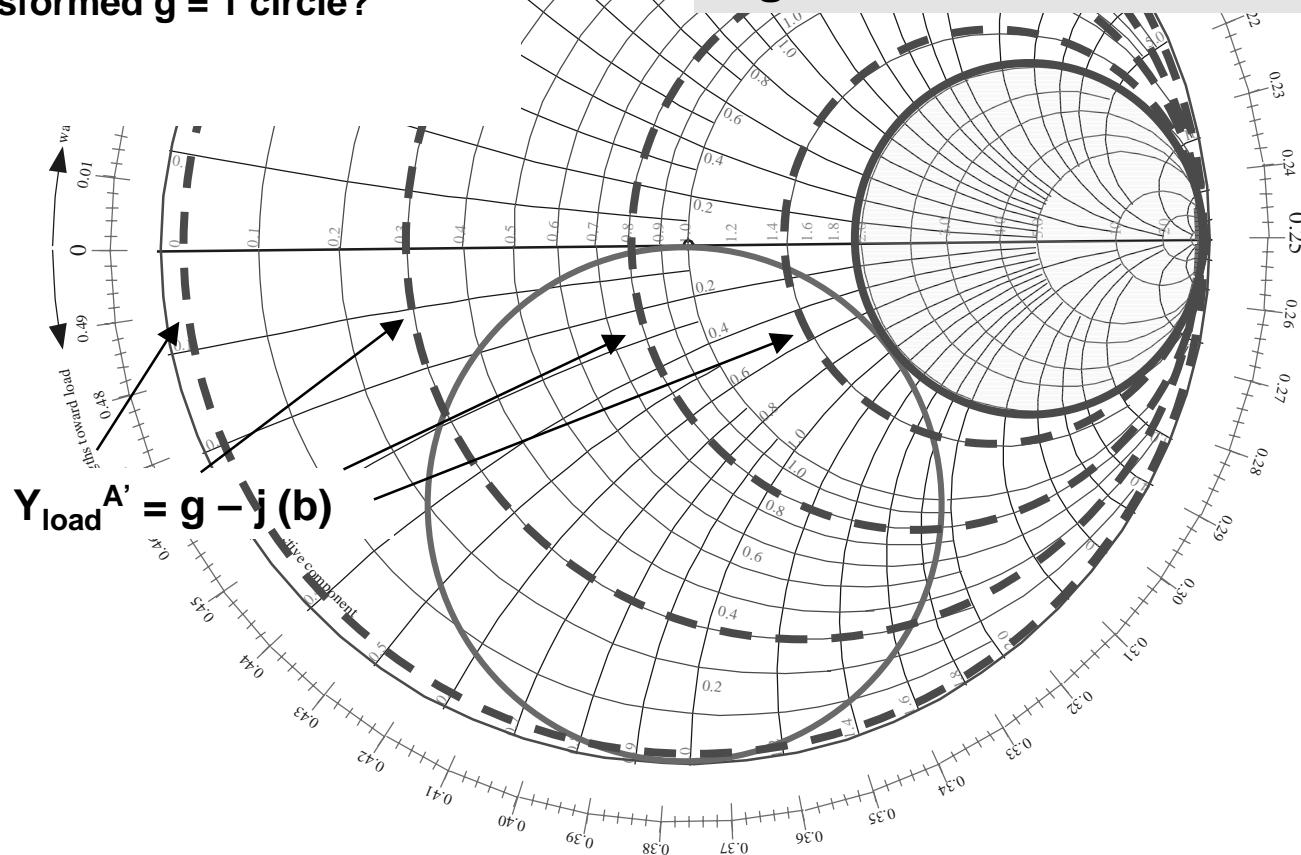
$$\Rightarrow l_1 = 0.102 \cdot \lambda$$

## “Forbidden” regions

- are there any values for  $Y^{A'}$  that could not produce a match?
  - recall we stayed on a curve of constant  $\text{Re}(Y^{A'})$  until we hit the transformed  $g = 1$  circle
  - are there any circles of  $\text{Re}(Y^{A'})$  that do NOT intersect the transformed  $g = 1$  circle?

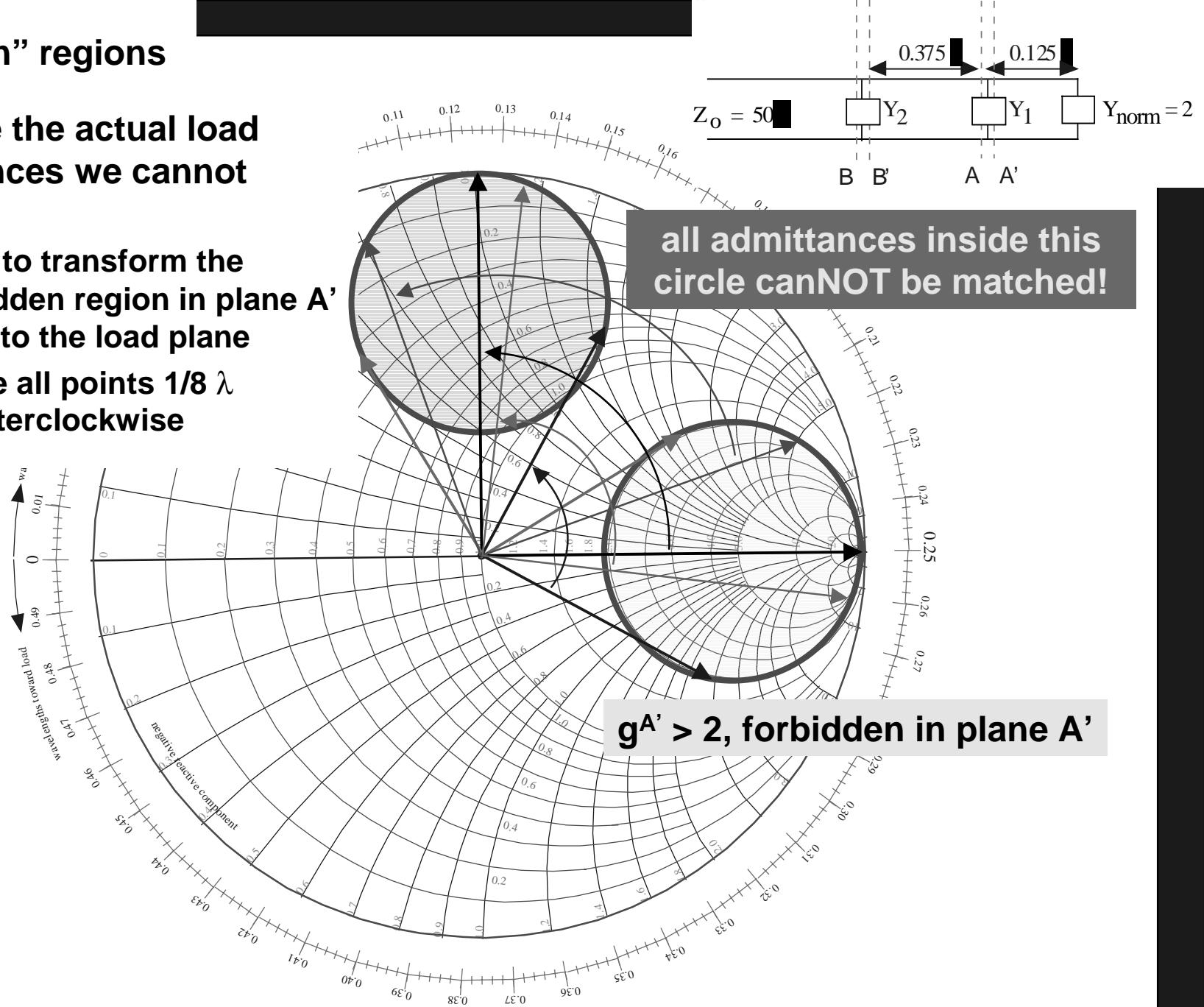


**if  $g^{A'} > 2$  there is NO intersection!**

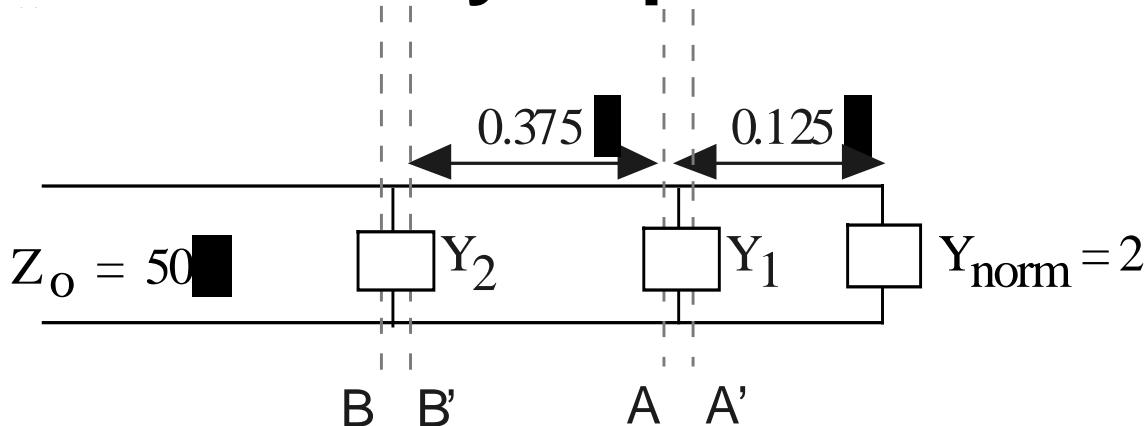


## “Forbidden” regions

- what are the actual load admittances we cannot match?
  - need to transform the forbidden region in plane A' back to the load plane
  - rotate all points  $1/8 \lambda$  counterclockwise



# summary of procedure

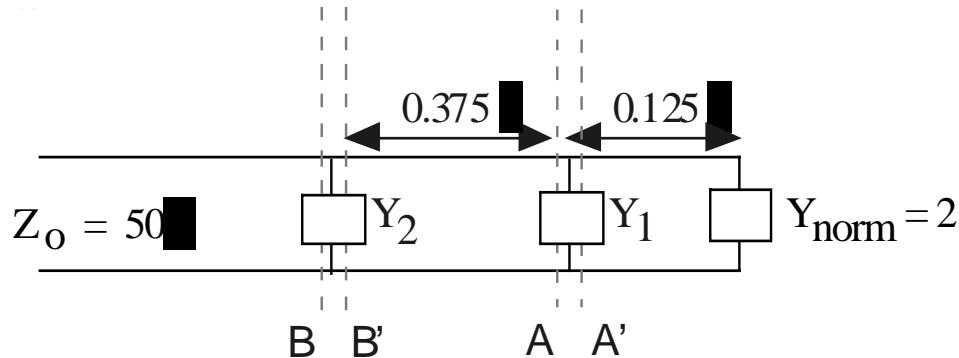


- move target in plane B ( $g=1, b=0$ ) to plane B':  $y = 1 + j^* \text{something}$ 
  - $g = 1$  circle
- move  $g = 1$  circle in plane B' to plane A
  - rotate whole circle appropriate distance
- move load to plane A'
- add susceptance Y<sub>1</sub> to move onto transformed  $g = 1$  circle in plane A
- move from plane A back to plane B'
  - if you did it right you will now be on the  $g = 1$  circle!
- add susceptance Y<sub>2</sub> to move along  $g = 1$  circle to  $b = 0$
- DONE (except for finding lengths to go with Y<sub>1</sub> and Y<sub>2</sub>)

# Solve numerically

- use the “tangent equation” to calculate the admittances

$$Y_{in}\left(Y_O, Y_{load}, 2\pi \cdot \frac{l}{\lambda}\right) = Y_O \cdot \frac{Y_{load} + j \cdot Y_O \cdot \tan\left(2\pi \cdot \frac{l}{\lambda}\right)}{Y_O + j \cdot Y_{load} \cdot \tan\left(2\pi \cdot \frac{l}{\lambda}\right)}$$



$$Y_{A'} = Y_{in}\left(Y_O, 2 \cdot Y_O, 2\pi \cdot \frac{1}{8}\right)$$

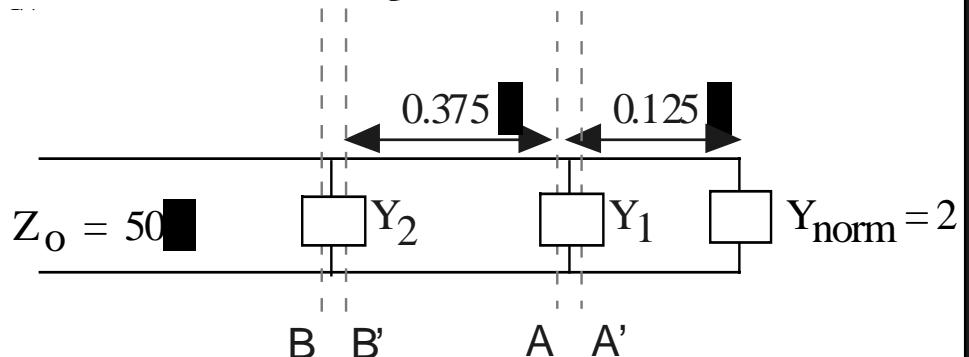
$$Y_1 = Y_{in}\left(Y_O, \infty, 2\pi \cdot \frac{l_1}{\lambda}\right)$$

$$Y_A = Y_1 + Y_{A'}$$

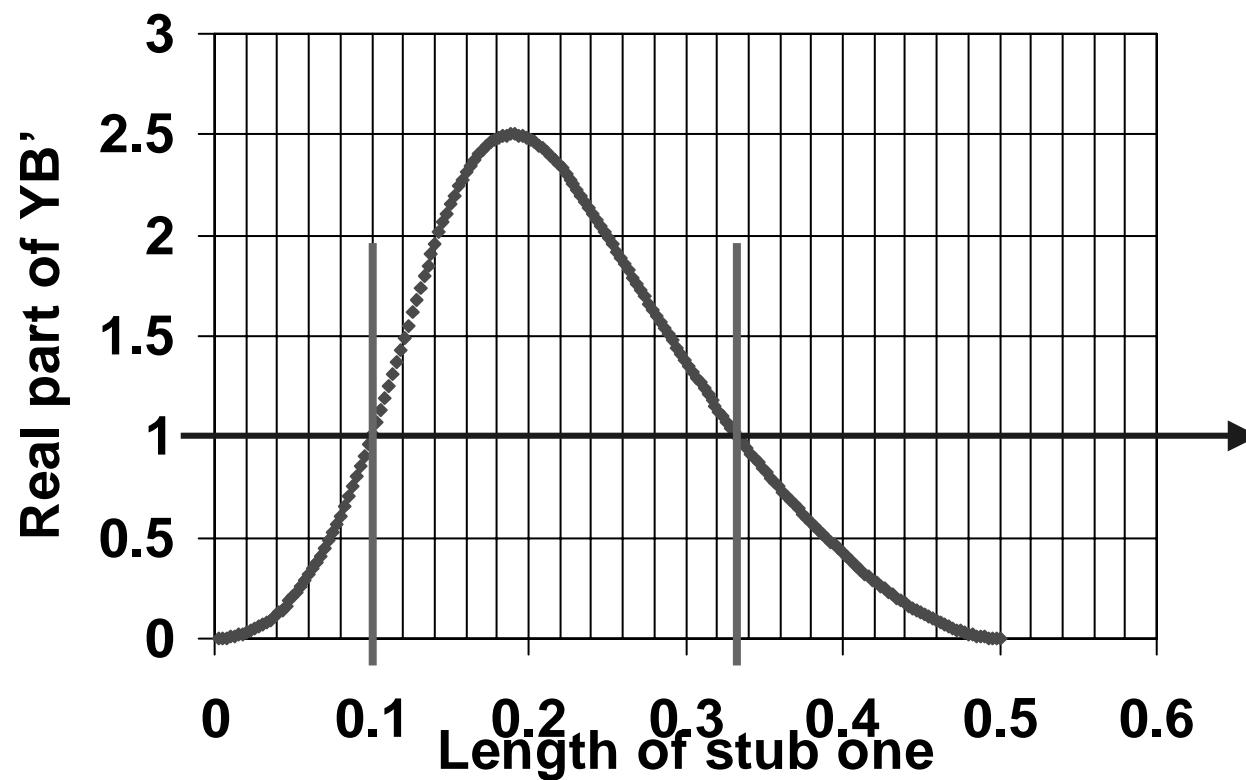
$$Y_{B'} = Y_{in}\left(Y_O, Y_A, 2\pi \cdot \frac{3}{8}\right)$$

- find  $l_1 / \lambda$  such that  $\text{Re}(Y_{B'} / Y_O) = 1$

# Solve numerically



- real part of  $Y_B'$ , normalized to  $Y_o$ , as a function of  $l_1 / \lambda$  :



$$\frac{l_1}{\lambda} = \begin{cases} 0.100 \\ \text{or} \\ 0.334 \end{cases}$$