How high/fast can we go??

- Limits set by:
  - electromagnetics and relativity
  - material properties
  - fabrication capabilities
  - "PARASITICS"
- In a complete, useful "system," you have to worry about everything!
  - "performance" is a hard thing to quantify

All of electromagnetics in one slide

- **Maxwell's equations, differential form:**

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M} \qquad \nabla \bullet \vec{D} = \rho$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad \nabla \bullet \vec{B} = 0$$

- **constitutive relations (material properties):**

$$\vec{B} = \mu \vec{H} \qquad \vec{D} = \varepsilon \vec{E} \qquad \vec{J} = \sigma \vec{E}$$

So what??

- **Wave equations**
  - **source-free, linear, isotropic, homogeneous region, assuming time dependence is $e^{j\omega t}$:**

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

  - **which gives the Helmholtz eq.:**

$$\nabla^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0$$

  - **the wavenumber, or propagation constant for the wave solution to this differential eq. is:**

$$k = \omega\sqrt{\mu\varepsilon}$$

  - **phase and group velocities for this case are:**

$$v_p = v_g = \frac{1}{\sqrt{\mu\varepsilon}}$$

What happens in a conductor?

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \sigma\vec{E}$$

$$\nabla^2\vec{E} + \omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\vec{E} = 0$$

- **you have a "dielectric" when:**

$$\frac{\sigma}{\varepsilon} << \omega$$

- **and a conductor if:**

$$\frac{\sigma}{\varepsilon} >> \omega$$

"Semiconductor" (i.e., material) equations:

- **Poisson's equation:** $\nabla \bullet \vec{D} = \rho$
- **continuity eq.:** $\nabla \bullet \vec{J} + \frac{\partial \rho}{\partial t} = 0$
- **"Ohm's" law:** $\vec{J} = \sigma \vec{E}$
- **but what is σ??**
  - **typically we use:** $\sigma = q \mu n$
  - **where μ is the "mobility"** $\mu = \frac{velocity}{E}$

Questions so far:

- How big can σ be?
  - how big can μ be?
  - how high are velocities?
- How big are the dielectric relaxation frequencies?

What affects the "mobility" in a semiconductor?

- We need to look at "band structure" first
- consider a free charged particle

  - kinetic energy E $E = \frac{p^2}{2m}$
  - momentum - wave vector relation $p = \hbar k$
  - energy-wave vector relation E(K) $E = \frac{\hbar^2}{2m} \cdot k^2$
  - first derrivative of E(k)

$$\frac{dE}{dk} = \frac{\hbar}{m}(\hbar k) = \frac{\hbar}{m}(p) = \frac{\hbar}{m}(mv) = \hbar v \quad \Rightarrow \quad velocity = \frac{1}{\hbar}\left(\frac{dE}{dk}\right)$$

  - second derrivative

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m} \quad \Rightarrow \quad m = \hbar^2 \Big/ \frac{d^2E}{dk^2}$$

Free particle under influence of constant Electric Field

- **force**

$$\vec{F} = q\vec{E}$$

- **Newton's second law**

$$m\frac{d^2x}{dt^2} = qE$$

- **integrate to get velocity, starting at rest**

$$v = \frac{q}{m}E\,t$$

- **current density $\propto$ velocity**

$$J \propto \left(\frac{q}{m}t\right)E$$

- **doesn't look like Ohm's law unless "conductivity" is linearly increasing with time!**

Are charged carriers in a material free?

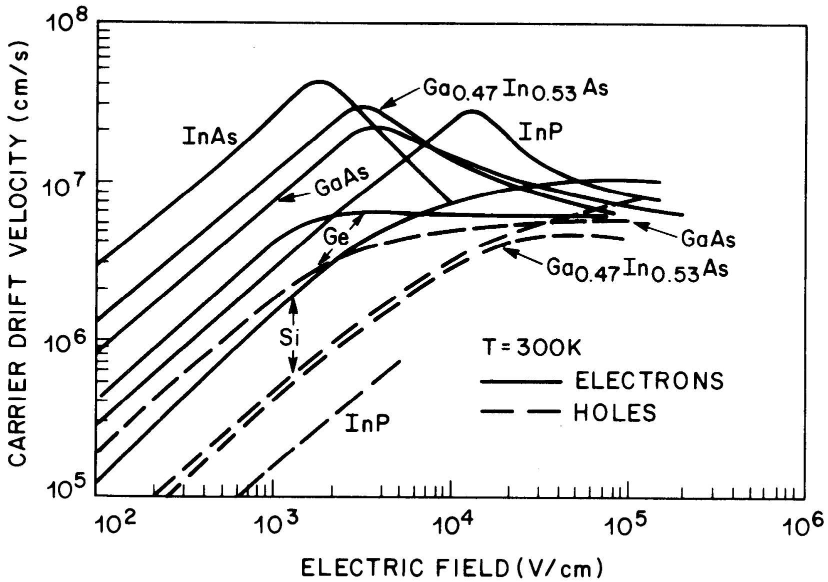
- **NO!**
  - **"particles" collide; are not freely accerlerated**
  - **if there are randomizing collisions, with a mean time between collisions of τ, the mean "drift" velocity is**

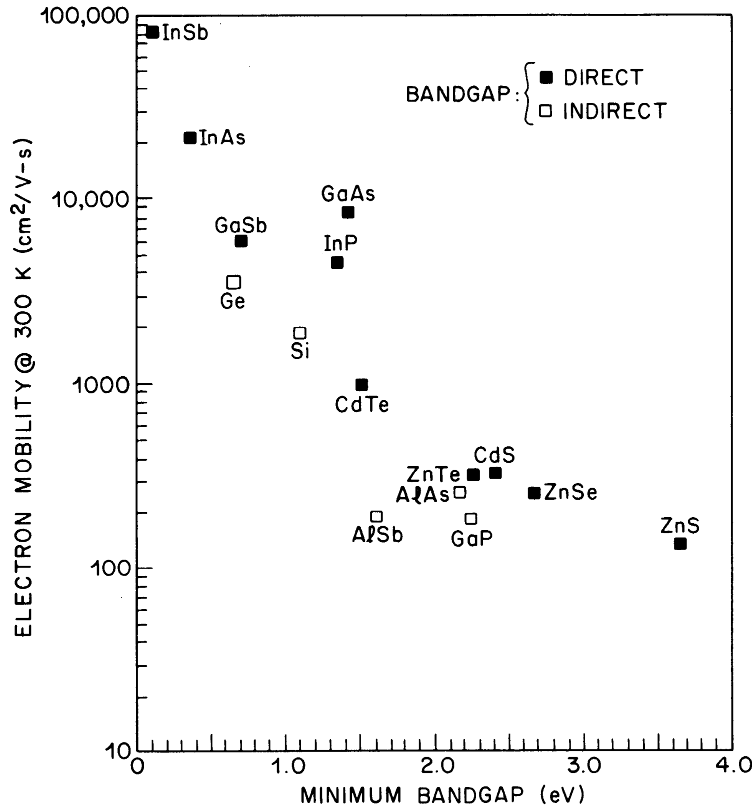
$$v = \frac{q}{m} E \left(\frac{\tau}{2} \right)$$

  - **looks like a "free" particle with time of flight τ /2**

What happens in a periodic crystal?

- quantum mechanical solution in terms of "waves"
- periodicity induces "band structure"
  - generally shown as E(k) relations
- "particles" are "free" in a band, made up of a superposition of wavefunctions from that band
  - effective mass $m^* = \hbar^2 \Big/ \frac{d^2E}{dk^2}$
  - mobility $\mu = \frac{v}{E} = \frac{q\tau}{2m^*}$
- want low m* and long "relaxation" time



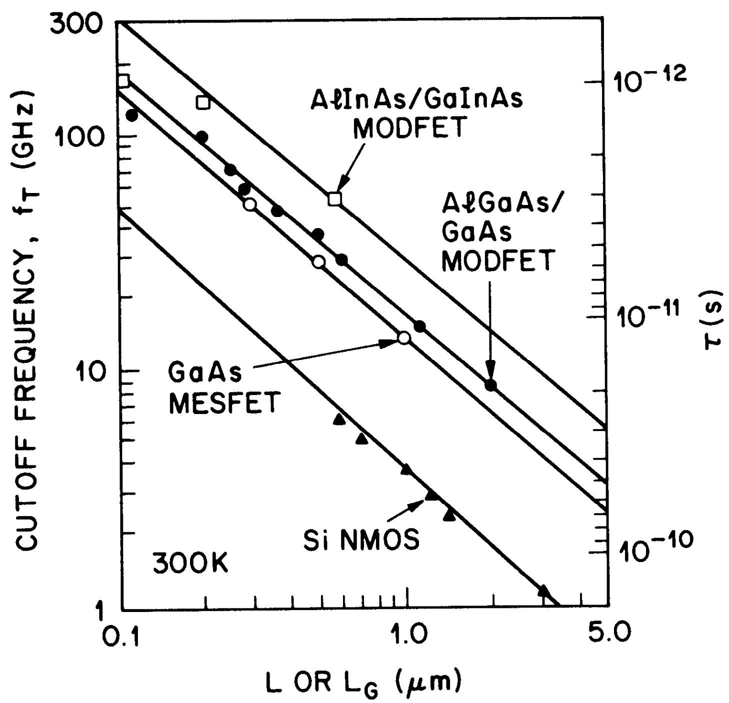


- from: S. M. Sze, “High-Speed Semiconductor Devices,” . New York: John Wiley & Sons, Inc., 1990, p. 18.

•from: S. M. Sze, “High-Speed Semiconductor Devices,” . New York: John Wiley & Sons, Inc., 1990, p. 15.

How do we fabricate fast things?

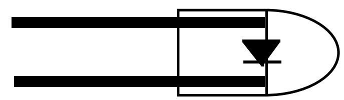
- **Make it SMALL!!**
- **Silicon or GaAs?**
  - **silicon is a much more mature technology**
    - **better processing**
    - **much higher levels of integration**
  - **GaAs has somewhat "better" electronic properties**
    - **higher mobilities**
    - **high v_{sat}'s**
    - **mechanical and thermal properties are worse**
- **BUT 99% OF THE PERFORMANCE IS IN HOW SMALL YOU MAKE THE DEVICE!**



- from:S. M. Sze, "High-Speed Semiconductor Devices," . New York: John Wiley & Sons, Inc., 1990, p.5.

And what are parasitics?

- consider a diode with wires attached



- how do you find the "impedance" of this device at high frequencies/speeds?
- does the "package" matter?
- is the device dominated by "intrinsic" device physics or by its "extrinsic" connection to the rest of the world?

"Packaged" diode model

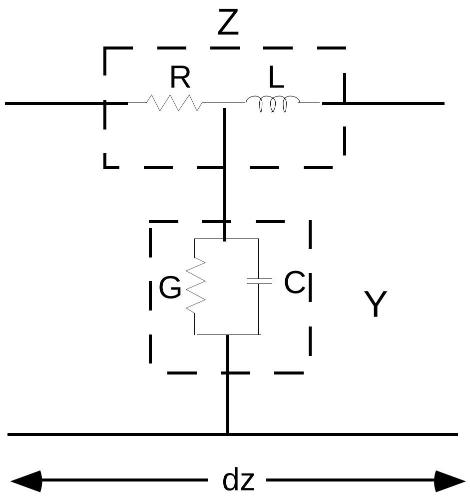
- need two things
  - "device" model
    - equivalent circuit
    - 5 µm diameter diode, 0.1 µm depletion width
    - forward bias assume 10^5 amps/cm^2 at 0.1 V
    - reverse bias assume "open"
    - parasitic contact resistance 5×10^{-7} cm^2
  - "package" model?

"package" model: transmission line?

- transmission lines are "distributed" systems
  - whenever the size of the circuit is large compared to the appropriate scale length (the electromagnetic wavelength here) the system cannot be represented by a single "lumped" circuit element
  - generalized model in terms of infinitesimal circuit
  - in time harmonic (e^{jwt}) case get "telegraphist's equations":

$$\frac{dV}{dz} = -Z\,I \qquad \frac{dI}{dz} = -Y\,V$$

$$Z = R + j\omega L \qquad Y = G + j\omega C$$



Traveling wave solutions for T-lines

$$V = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \qquad I = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$

- **propagation constant γ:** $\gamma = \sqrt{Z\,Y}$
- **characteristic impedance Z_o:** $\frac{V_o^+}{I_o^+} = Z_o = -\frac{V_o^-}{I_o^-} = \sqrt{\frac{Z}{Y}}$
- **for "low" loss or "high" frequency:**

$$Z \approx j\omega L \qquad Y \approx j\omega C$$

$$\gamma = j\beta = j\omega\sqrt{LC} = j\omega\sqrt{\mu\varepsilon} \qquad Z_o = \sqrt{\frac{L}{C}}$$

"Input" impedance of a terminated T-line

- **impedance is a function of both position and load:**

$$Z_{in}(\ell) = Z_o \frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell}$$

- **reflection coefficient is a convenient number:**

$$\Gamma = \frac{V_-}{V_+} = \frac{Z - Z_o}{Z + Z_o} \qquad \Gamma(0) = \frac{Z_L - Z_o}{Z_o + Z_L}$$

- **for a lossless line both are simple periodic functions:**

$$Z_{in}(\ell) = Z_o \frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell} \qquad \Gamma(\ell) = \Gamma(0) e^{-2j\beta\ell}$$

$$\beta = j\omega\sqrt{LC} = \frac{2\pi}{\lambda}$$