Accurate Time Domain Multiconductor Lossy Transmission Line Analysis Including the Skin Effect

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Abstract

A new method for time domain waveform evaluation on finite conductivity multi-conductor transmission lines is presented. An equivalent circuit model directly determined from conductor geometry is used, giving a lossless-like equation. Combined with the method of characteristics, very fast and accurate waveform calculations result.

Introduction

As clock speeds become faster and chips become more complex, it is more difficult to accurately predict delay and crosstalk in densely packed multi-conductor interconnects. The impact of frequency dependent inductance and resistance due to finite conductivity conductors on time domain waveforms has often been ignored or only poorly approximated even though this can cause overestimation of far-end crosstalk and underestimation of delay. To include frequency dependent effects accurately in time domain calculations, transformation using FFT is usually required. However, it is difficult to implement non-linear drivers using this method. In [3], a surface impedance boundary condition was applied directly to the time domain to solve radiation problems where high frequency approximation for surface impedance boundary condition is accurate enough. Here we show a new method for transmission line analysis where the boundary condition is represented directly in the time domain. Unlike other time domain simulation methods in which all the RLC parameters of the transmission lines must be provided a priori, in our model all required values are determined directly and simply from conductor geometry. The resulting frequency domain model is valid over a wide frequency range, and hence giving accurate time domain results. The method is also compatible with non-linear drivers and loads.

Application of impedance boundary condition in the time domain

Rapid calculation of interconnect series impedance in the frequency domain can be performed using an impedance boundary condition [2]. For a rectangular conductor cross section, the cross section is partitioned into two trapezoidal and triangular parts as shown in figure 2a; the “transverse impedance” of each part is used as an approximation of impedance boundary condition for the conductor [2]. To complete the problem, each transverse impedance is assigned to a corresponding surface ribbon and mutual inductances between these ribbons are calculated. The resulting two dimensional equation is

$$\left[Z_{eii}\right][I]+j\omega[L][I]=-\frac{\partial}{\partial z}[V] \quad , \qquad (1)$$

where $\left[Z_{eii}\right]$ is a diagonal matrix of the effective internal impedances (EII) (representing the impedance boundary condition) and $[L]$ is a dense matrix of mutual inductances between ribbons.

To use this frequency domain concept in time domain analysis, the equivalent circuit shown in figure 2b is used to model each ribbon. This produces a rational function in the s-domain that is easily converted into time domain exponential functions. All values in this circuit model are determined directly from simple, closed form expressions depending on only the conductivity and geometry of the conductors [5, 6]. Transforming (1) into the time domain gives

$$[\zeta(t)] * \frac{\partial}{\partial t}[I] + [L]\frac{\partial}{\partial t}[I] = -\frac{\partial}{\partial z}[V] \quad , \qquad (2)$$

where '*' represents convolution, $[\zeta(t)]$ is a sum of exponential functions obtained by converting the rational function in s-domain $[Z_{eii}/s]$, i.e.,

$$\frac{Z_{eii}}{s} = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s\left(b_3 s^3 + b_2 s^2 + b_1 s + b_0\right)} = \frac{k_1}{s} + \frac{k_2}{s+p_2} + \frac{k_3}{s+p_3} + \frac{k_4}{s+p_4} \approx \frac{k_1}{s} + \frac{K}{s+P} \quad . \qquad (3)$$

The rational function in (3) can be further reduced by using dominant pole approximation to reduce the computation time and memory usage.

The convolution appearing in (2) can be evaluated easily using a recursive relationship [7],

$$Y(n\Delta t) = ke^{p\Delta t} \cdot X(n\Delta t) = k\Delta t \cdot X(n\Delta t) + e^{p\Delta t} Y((n-1)\Delta t) \quad . \qquad (4)$$

Applying (3) to (2) yields

$$([k\Delta t] + [L])\frac{\partial}{\partial t}[I] + [V_{ds}] = \left[L^{'}\right]\frac{\partial}{\partial t}[I] + [V_{ds}] = -\frac{\partial}{\partial z}[V] \quad , \qquad (5)$$

where $[V_{ds}]$ is an extra voltage-dependent source added to the telegrapher's equation, depending on Δt and on the poles and residues of the equivalent circuit of the EII. Since Δt is usually very small, this voltage dependent source is mainly responsible for incorporating the current redistribution effects (i.e., the frequency dependent proximity and skin effects) in the time domain. Equation 4 is then solved along with the second telegrapher's equation for the shunt admittance (e.g., the capacitance) of the interconnect.

Method of characteristics

The method of characteristics was first developed to simulate a single lossless line [8], then later extended to multi-resistive lines [9]. Recently, it was further extended to non-uniform lossy line cases [10], where frequency dependent series impedance is calculated using FFT. Here, for simplicity, a homogeneous medium with $[G]$=0 is assumed. More general cases are well demonstrated in [10]. From equation 4 and the corresponding admittance equation,

$$d([V] \pm [R][I])/dt = -[V_{ds}] \cdot v_p \quad , \qquad (6)$$

where $[R] = v_p\left[L^{'}\right]$, v_p is propagation velocity and +/- represent incident and reflected waves, respectively. To capture the distributed resistive effects intermediate segmentation is required, then equation 8 becomes

$$\{[V] + [R][I]\}(x_{k+1}, t_{n+1}) = \{[V] + [R][I] - \Delta t \cdot v_p [V_{ds}]\}(x_k, t_n) \ , \qquad (7a)$$

for the incident wave, and for the reflected wave

$$\{[V] - [R][I]\}(x_{k-1}, t_{n+1}) = \{[V] - [R][I] + \Delta t \cdot v_p [V_{ds}]\}(x_k, t_n) \quad . \qquad (7b)$$

The equivalent model is shown in figure 3; comparing to the model shown in [11], the resistance is replaced with a new voltage-dependent source to capture frequency dependent effects. To simulate rectangular conductors, each conductor is divided into four parts as shown in figure 2a, then each part is represented with its equivalent circuit, and finally four parts are connected in parallel.

Results and conclusions

Figure 4 shows simulation results obtained using the new method introduced in this paper, as well as illustrated the errors resulting from failure to properly include the time/frequency dependencies of both resistance and inductance. Excellent agreement is shown compared to FFT method using a full dispersion curve, while the CPU time required for this method (7.6 seconds) is much less than the FFT method (283 seconds). In conclusion, this paper demonstrates that the dc to skin effect frequency variation of series impedance can be directly and accurately included in the time domain, with all needed values can be easily determined directly from conductor geometry, while maintaining high numerical efficiency.

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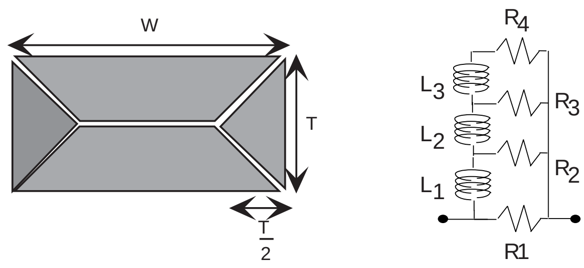


Figure 2) (a) Segmentation scheme for rectangular bar to approximate surface impedance. (b) Equivalent circuit model for approximated surface impedance.

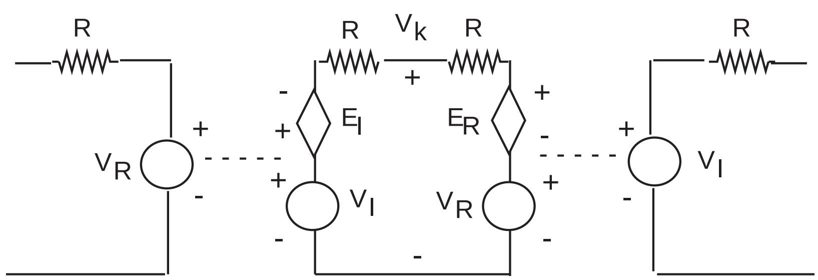
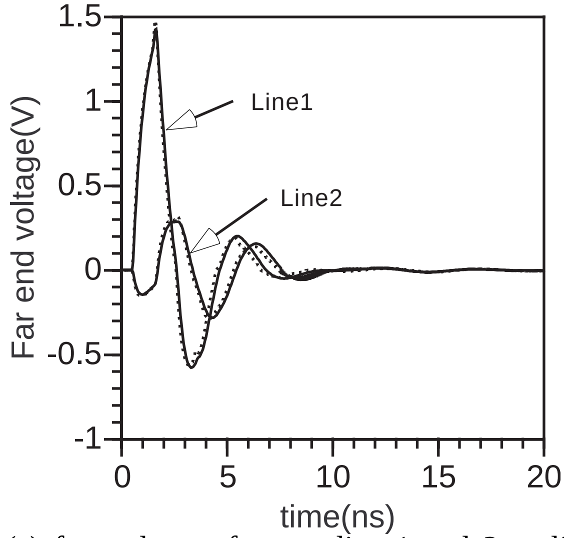
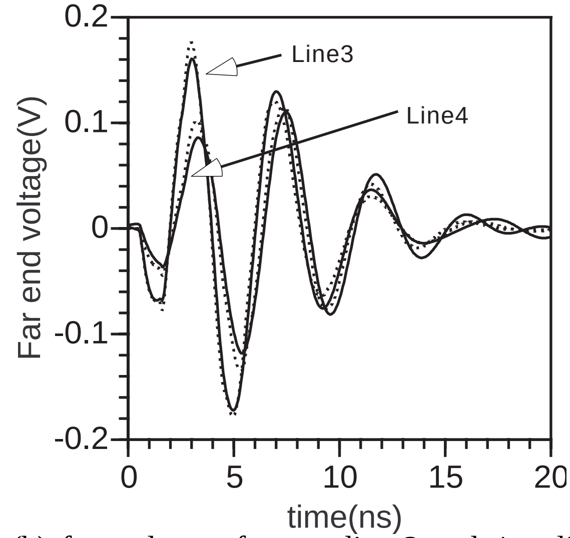


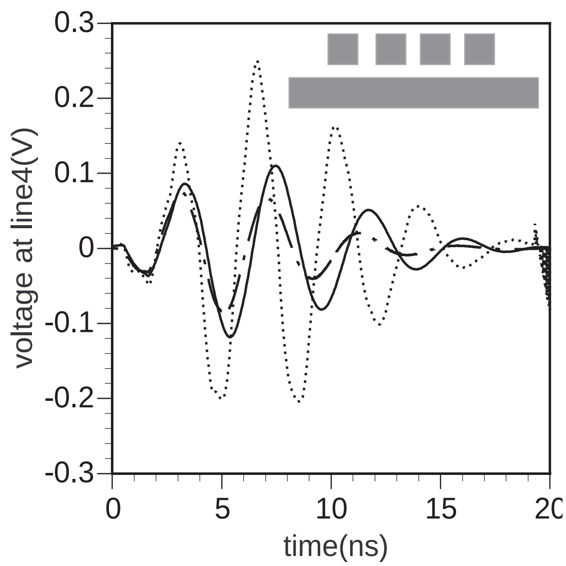
Figure 3) Equivalent model for method of characteristics. E_R and E_I are voltage dependent sources of reflected and incident waves respectively and appear in second part of right hand side of the equation 8a and 8b, respectively.



(a) far end waveform at line 1 and 2; solid line: FFT result, dotted line: new method described here.



(b) far end waveform at line 3 and 4; solid line: FFT result, dotted line: new method described here.



(c) far end voltage waveform at line 4: errors induced by failure to correctly include both resistance and inductance frequency dependencies: Solid line: FFT using full dispersion curve; dotted line: constant R-L-C transmission line analysis; dashed-dotted line: Star-Hspice® 97.2 using W Element where one high frequency resistance value must be calculated using some other method (here, the surface ribbon method was used [2]).

Figure 4) Simulation results for 4 lines above a ground plane. Conductor geometry: width & thickness = 20 μm, separation between conductors = 10 μm, signal conductors 10 μm above ground plane, ground plane 20 μm thick, 170 μm wide, line length = 0.1 m. Input wave form applied to line 1 only: 1V trapezoidal wave with rise, fall time = 0.1 ns, duration = 1 ns; source resistance = 5 Ω, terminated with 10 pF capacitance.