Chapter 2

Optical characteristics of Fabry-Perot cavity sensors

As a sensor, a Fabry-Perot cavity can transform some measurands, such as force and temperature, into changes in the optical response of the cavity. For example, the reflectance (or transmittance) of a Fabry-Perot cavity varies as the measurands change the optical path length between the two mirrors of the cavity. In this chapter optical characteristics of a micromachined Fabry-Perot cavity with multiple layers are calculated using the characteristic matrix method. Using the method the reflectances of micromachined Fabry-Perot cavity sensors are calculated as a function of the gap between the two mirrors.

2.1 WAVE PROPAGATION THROUGH MULTIPLE LAYERS

When a wave is propagating from one medium to another medium, reflection of the incident wave always occurs at the interface between the two media. The reflection is necessary to satisfy the boundary conditions at the interface and Maxwell's equations. The reflectance (R) is defined as a ratio of reflected power to incident power, given by

$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2, \tag{2.1}$$

where n_1 and n_2 are refractive indices of each medium, assuming the wave propagates from medium 1 to medium 2. Transmittance at the interface becomes $1 - R$.

For a multiple film stack, calculation of the overall reflectance and transmittance becomes very complicated because multiple reflections occur at every interface in the film stack. To calculate the optical characteristics of the multiple film stack the characteristic matrix method can be used. In this method each film is represented by a matrix, called characteristic matrix. The characteristic matrix includes effects due to the thickness and the refractive index of each film. The optical characteristics of the multiple film stack can be calculated by using a product of all the characteristic matrices.

Figure 2.1 shows a schematic view of a multiple film stack, e.g., a Fabry-Perot cavity, consisting of q layers, where for convenience the air gap (of length g) is always labeled as the k^{th} layer. Each layer has refractive index n_i which can be complex, $n_i = n_i' - j\,k_i'$, to represent a lossy medium. From the refractive index the admittance of the layer is

$$\eta_i = \mathrm{Y}_o \cdot n_i,$$

where Y_o is the admittance in vacuum (1/377 siemens). Suppose a plane wave with wavelength (λ_o) in vacuum is propagating into the Fabry-Perot cavity at normal incidence. The characteristic matrix M_i of an optically homogenous layer with refractive index n_i and thickness z_i is

$$\mathrm{M}_i = \begin{pmatrix} \cos\left(k_o\, n_i\, z_i\right) & \dfrac{j}{\eta_i}\sin\left(k_o\, n_i\, z_i\right) \\ j\,\eta_i\,\sin\left(k_o\, n_i\, z_i\right) & \cos\left(k_o\, n_i\, z_i\right) \end{pmatrix}, \tag{2.2}$$

where $k_o = 2\pi / \lambda_o$ [1] .

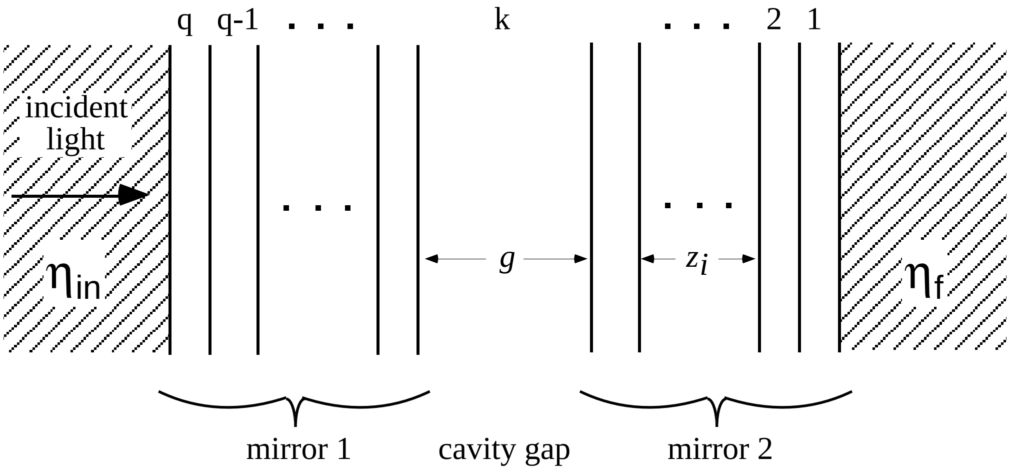

Figure 2.1 : Schematic diagram of a multilayer Fabry-Perot sensor.

For a multiple film stack as shown in Figure 2.1, the equivalent characteristic matrix M of the film stack can be calculated using

$$\mathrm{M} = \mathrm{M}_q \cdot \mathrm{M}_{q-1} \cdots \mathrm{M}_2 \cdot \mathrm{M}_1 . \tag{2.3}$$

Using the equivalent characteristic matrix M the reflectance R and the transmittance T of this structure are calculated using

$$R = \left| \frac{\eta_{in} B - C}{\eta_{in} B + C} \right|^2 \tag{2.4}$$

$$T = \frac{\left| 2\eta_{in} \eta_f \right|}{\left| \eta_{in} B + C \right|^2}, \tag{2.5}$$

where

$$\begin{bmatrix} B \\ C \end{bmatrix} = \mathrm{M} \cdot \begin{bmatrix} 1 \\ \eta_f \end{bmatrix}.$$

η_f and η_{in} are the admittances of the environment outside the sensor and of the medium from which the light comes, respectively. In addition, the absorptance A of the structure is obtained from

$$A = 1 - (R + T). \tag{2.6}$$

The refractive index of a dielectric material is usually a function of wavelength. For example, the refractive indices of silicon dioxide and silicon nitride are shown in Figure 2.2 [2] .

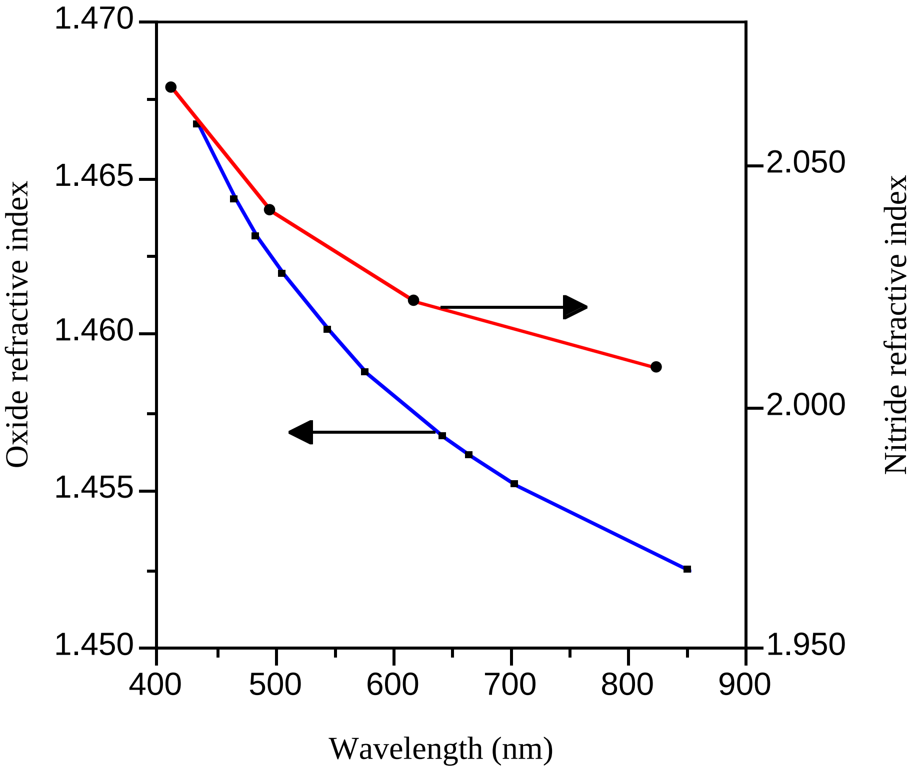

Figure 2.2 : Refractive indices of dielectric materials commonly used in silicon process (from ref. [2]).

To verify the model, a Fabry-Perot cavity with multiple dielectric films was fabricated and the transmittance of the cavity was measured as shown in Figure 2.3. For the top and bottom mirrors, a multiple film stack was used which consisted of two 1000 Å silicon nitride layers, one on each side of a 1400 Å silicon dioxide layer. All the layers were deposited by LPCVD (low pressure chemical vapor deposition). The silicon substrate under the cavity was anisotropically etched using KOH to allow transmittance measurement of the cavity. The transmittance of the cavity was measured using a spectrometer (Perkin Elmer Lambda 9) as well as calculated using the characteristic matrix method. The refractive index of the polysilicon layer was obtained from [2] . The measurement was in good agreement with the simulation except near resonant frequencies. The discrepancy near resonant frequencies is believed to result from inaccuracy of the imaginary part of the refractive index of polysilicon used from the reference [2] .

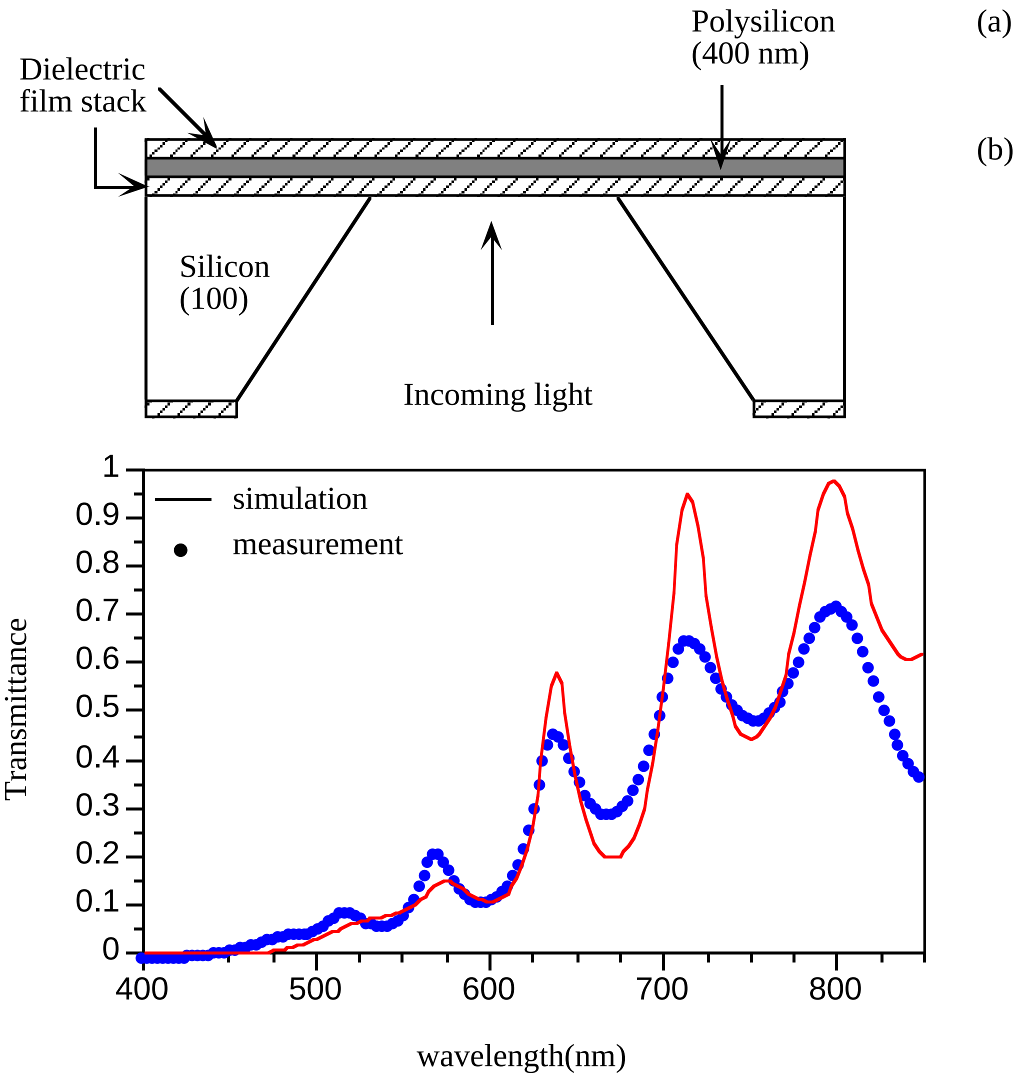

Figure 2.3 : Schematic view and transmittance of a Fabry-Perot cavity with a polysilicon spacer. (a) Both top and bottom dielectric film stacks consist of two 1000 Å silicon nitride layers cladding a 1400 Å silicon dioxide layer.

2.2 OPTICAL RESPONSE OF A FABRY-PEROT CAVITY PRESSURE SENSOR

With the characteristic matrix method, the optical response of a Fabry-Perot cavity sensor could be simulated as a function of a measurand, such as pressure or chemical reaction. In this section a micromachined Fabry-Perot cavity sensor, which is sensitive to differential pressure, is considered. When differential pressure is applied to the sensor, one of the mirrors in the cavity bends due to the pressure, leading to a change in the gap between the two mirrors, and in turn a change in the optical response.

Figure 2.4 illustrates a schematic cross sectional view of a micromachined Fabry-Perot cavity pressure sensor. To build such a device, a silicon wafer and a glass could be fusion-bonded to form a cavity. The silicon wafer would then be etched to form a pressure sensitive diaphragm which is typically a few microns thick. Dielectric or metal films could be deposited on both the silicon and the glass to give proper reflectivity of the mirrors. Using techniques similar to those described above, micro Fabry-Perot cavity pressure sensors have been fabricated [3, 4] . The sensors were usually used with optical fiber interconnects for remote sensing.

Calculation of reflectance of a cavity coupled to an optical fiber is not trivial because guided modes inside a fiber and coupling efficiency between a fiber and a cavity are very hard to calcuate. For simplicity of calcualtion, we assume a single mode fiber is used as an interconnect. Since the guided mode of a single mode fiber can be accurately approximated with only transverse components [5] , the incident wave to the cavity is assumed to be a plane wave with propagation direction normal to the surface of the cavity. Marcuse [6] showed that the guided field in a single mode fiber has Gaussian profile and calculated the coupling efficiency between two fibers using the beam profile. The coupling efficiency can be applied to calculate the reflectance of a Fabry-Perot cavity which is coupled to a single mode fiber. In this study we assume that there is no tilt or offset between the fiber and the cavity. When the cavity length is smaller than the diameter of the core of the single mode fiber, the coupling efficiency should be unity even for a ray having multiple reflection [6] . Under the above conditions, the reflectance of the cavity coupled to a single mode optical fiber is equivalent to the reflectance of a plane wave propagating the cavity at normal incidence.

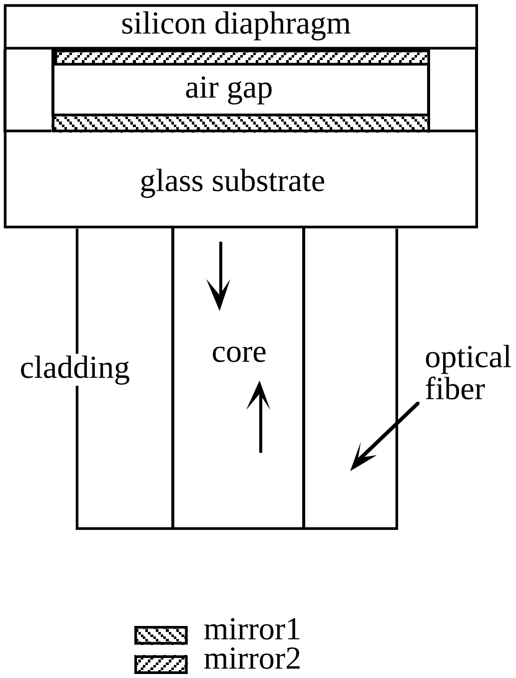

Figure 2.4 : Cross section of a Fabry-Perot cavity with a silicon diaphragm.

The change in the gap could be detected by monitoring the reflectance of the cavity. The reflectance of the cavity can be simulated as a function of the gap using the characteristic matrix method. For example, suppose each mirror of the cavity is made of a Au layer with nominal thickness of 70 (Å) which is deposited on both the silicon diaphragm and the glass substrate. The refractive index of gold (Au) is plotted as a function of wavelength in Figure 2.5 [2] .

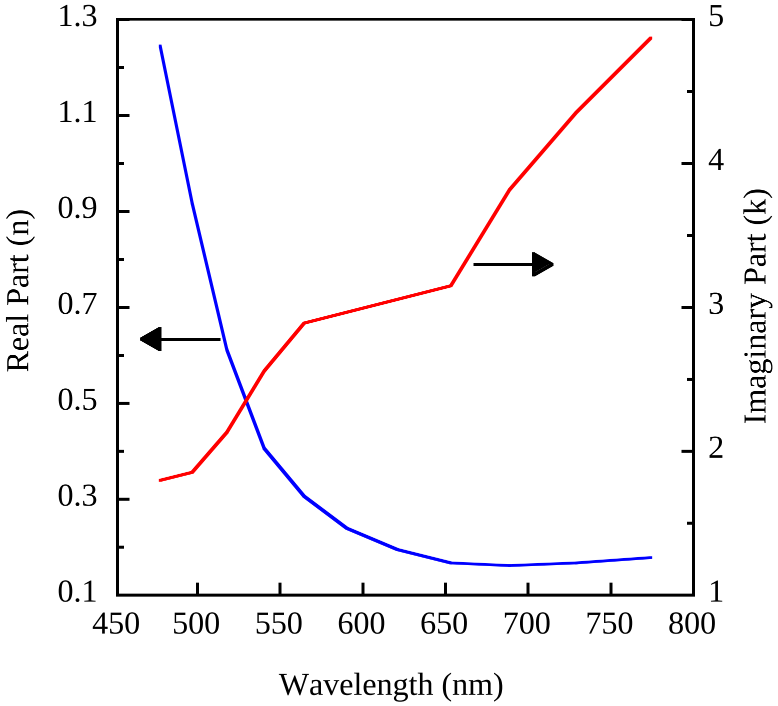

Figure 2.5 : Refractive index of gold versus wavelength (from ref. [2]).

Figure 2.6 shows a plot of the calculated reflectance of the cavity as a function of the gap, assuming an illumination wavelength of 700 nm. The reflectance has a period of $\frac{\lambda_o}{2}$. To avoid ambiguity of the gap measurement, the mechanical compliance of the diaphragm should be adjusted in such a way that the diaphragm would move only $\frac{\lambda_o}{4}$ over the a full range of pressure to be measured.

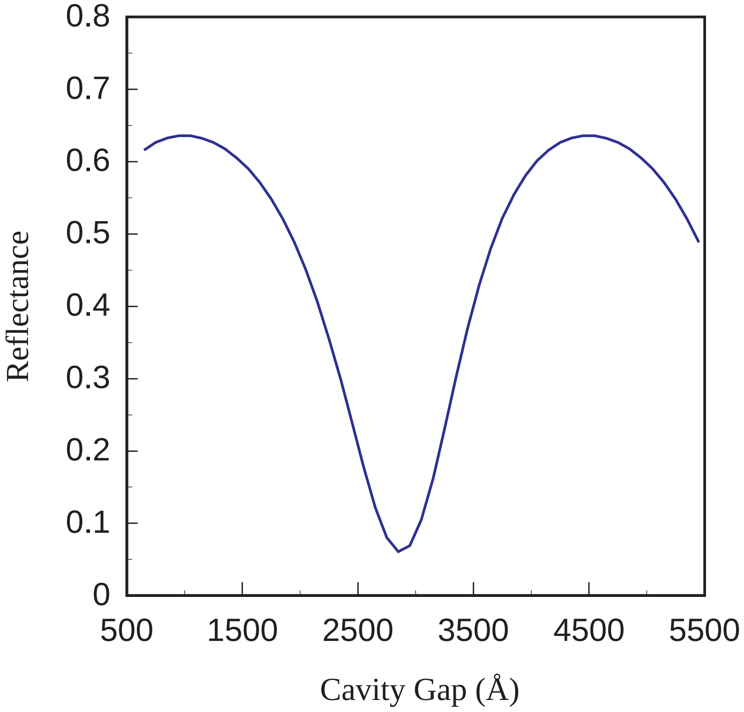

Figure 2.6 : Calculated reflectance of a Fabry-Perot cavity sensor with metal (Au) mirrors as a function of cavity gap.

A similar curve could also be obtained with multiple dielectric film mirrors, instead of metal film mirrors. By stacking several dielectric films with certain thickness, the reflectivity of the mirror could be adjusted at the operating wavelengths. Unlike the metal mirror, the reflectivity of the mirror is highly dependent on the wavelength.

As an illustration, Figure 2.7 shows a schematic view of a Fabry-Perot cavity sensor with dielectric film mirrors, while Figure 2.8 shows the reflectance of the cavity as a function of the gap. The top mirror is assumed to consist of two 1600 Å thick silicon nitride layers cladding a 4500 Å thick silicon dioxide layer. The bottom mirror is assumed to consist of two 600 Å silicon nitride layers cladding a 4200 Å silicon dioxide layer. In the calculation of the reflectance of the cavity, the space between the fiber and the bottom mirror is assumed to be filled with index matching fluid. A similar structure was demonstrated as a pressure sensor in [7] .

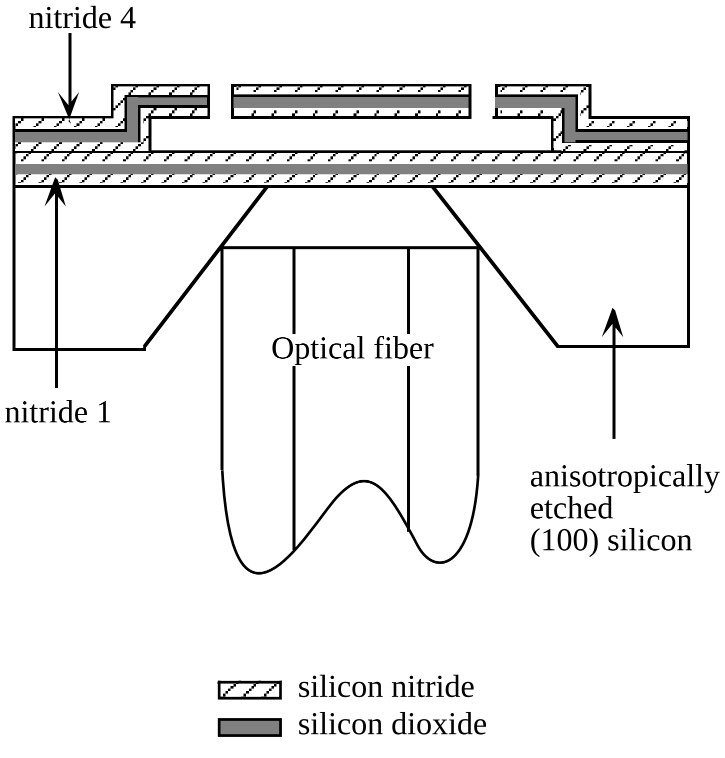

Figure 2.7 : Schematic cross sectional view of a Fabry-Perot cavity sensor with dielectric mirrors.

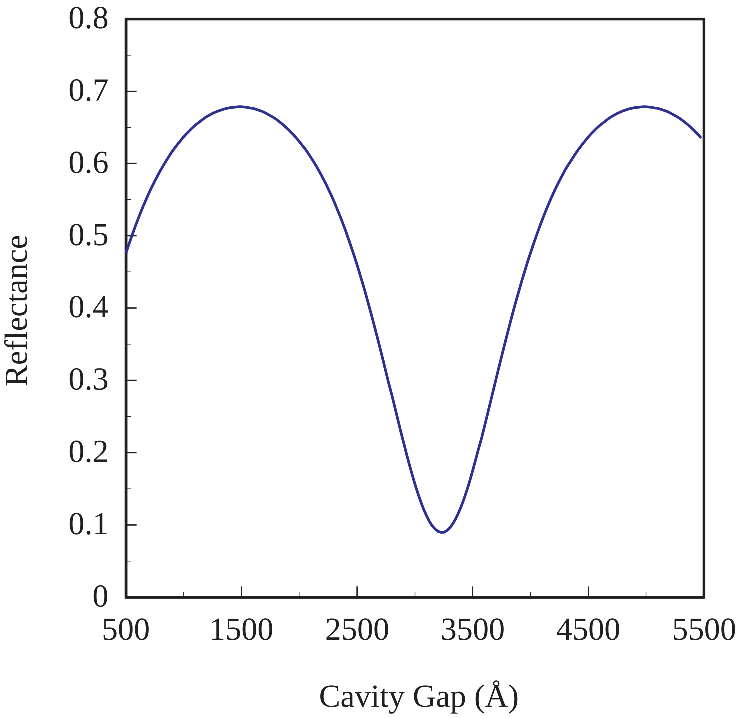

Figure 2.8 : Calculated reflectance of the sensor versus cavity gap, assuming an illumination wavelength of 700 nm.

2.3 SUMMARY

The optical characteristics of multilayer Fabry-Perot cavity sensors can be conveniently calculated using the characteristic matrix method. To verify the method a Fabry-Perot cavity with dielectric films was fabricated, and the transmittance of the cavity was measured at different wavelengths. The measurement agreed well with the calculation using the characteristic matrix method. The agreement implies that the films prepared by LPCVD, i.e. polysilicon, silicon nitride and silicon dioxide, could be used as mirror layers and the refractive indices of the films are consistent with values from reference [2] . With the method the reflectance of Fabry-Perot cavity sensors were also calculated as a function of the gap. The calculation simulated the change in the reflectance of the cavities when the pressure was applied to the sensors. The method will be later used to calculate variation of the optical response of multilayer cavities due to thickness variation of layers in the cavities in the next chapter.