

## **Chapter 4**

### **Deflection of multiple thin film diaphragms**

In this chapter mechanical properties of deposited films on silicon wafers are to be discussed. The properties, such as residual stress and Young's modulus, will be used to estimate deflection of films when external pressure is applied to them. The residual stress of a multiple film stack is measured by the beam curvature method. Also, the deflection of diaphragms, consisting of multiple dielectric films, is calculated by a numerical method, i.e., the Marcuse method.

#### **4.1 MECHANICAL PROPERTIES OF MULTIPLE FILM STACKS**

There are two mechanical properties of films to be considered when micromachined Fabry-Perot cavity sensors are built. They are the Young's modulus and the residual stress in the films, which determine mechanical compliance of the films and in turn the sensitivity of the sensors, and reliability of the process for the sensors. Since the properties of thin films are strongly affected by deposition conditions and subsequent processing steps, like annealing, the properties of films are difficult to predict at the design stage. The actual mechanical properties of films can be estimated by several measurement methods [1-5], after all the processing steps have been completed. Using micromachining techniques the methods allow the measurement of thin-film mechanical properties on micrometer scales, which can not be done with conventional methods. To monitor localized residual stress on wafers, microstructures have been placed on

the same wafers on which real devices are being fabricated [2-4] . In addition, novel methods, which extract both Young's modulus and stress of films at the same time, have been developed [1, 5] . With these methods Young's modulus and stress could be determined independently by analyzing load-deflection behavior of a diaphragm made of the films to be measured.

Most micromechanical structures have residual stress built up during fabrication due to intrinsic stress and thermal stress. The stress inside the structures tends to produce unwanted tension or compression forces and, in turn, causes buckling or cracking of the microstructures when the structures are released from the substrate. Deformation of the microstructures are undesirable for many applications, especially for Fabry-Perot cavities. To prevent the deformation, a composite film stack, consisting of films under compressive stress and under tensile stress, has been used for supporting microstructures. The composite stack is found to have less total stress by compensating the compressive (or tensile) stress of films with the tensile (or compressive) stress of other films.

Equivalent residual stress and Young's modulus of a composite film stack can be obtained using the equations [5]

$$E_{eq} = \frac{\sum_i (E_i \cdot h_i)}{\sum_i h_i} \quad \text{and} \quad \sigma_{eq} = \frac{\sum_i (\sigma_i \cdot h_i)}{\sum_i h_i}, \quad (4.1)$$

where  $E_i$  and  $\sigma_i$  are Young's modulus and residual stress of the  $i$ 'th film, respectively.

Figure 4.1 shows measurement of residual stress in dielectric films deposited by LPCVD (low pressure chemical vapor deposition) on Si substrates. The residual stress was estimated by measuring the curvature of the Si substrates before and after depositing films. For a given radius of curvature the stress ( $\sigma$ ) of the film is obtained from the following equations [6, 7] :

$$\sigma = \frac{E}{1-\nu} \cdot \frac{l_{sub}^2}{l_{film}} \cdot \frac{1}{6r} , \quad (4.2)$$

where  $l_{film}$  is the thickness of a film,  $l_{sub}$  is the thickness of the substrate,  $r$  is the radius of curvature, and  $\nu$  is Poisson's ratio of the substrate. By convention,  $r$  is negative for convex curvature induced by compressive stress and positive for concave curvature induced by tensile stress. As implied in equation (4.1), the measured residual stress of a composite film, consisting of silicon dioxide (compressive) and silicon nitride (tensile), is observed to be less than the stress of silicon nitride. The measured residual stress of the dielectric films agrees with previously published values [5, 8] .

Note that the stress of a film is a function of the deposition conditions of the film. Silicon nitride was deposited by reaction of ammonia (NH<sub>3</sub>) and dichlorosilane (SiCl<sub>2</sub>H<sub>2</sub>) at gas flow rate of 3.5:1 at 800 °C and 220 mTorr. Silicon dioxide was deposited by reaction of silane (SiH<sub>4</sub>) and oxygen (O<sub>2</sub>) at gas flow rate of 3:4 at 450 °C and 110 mTorr.

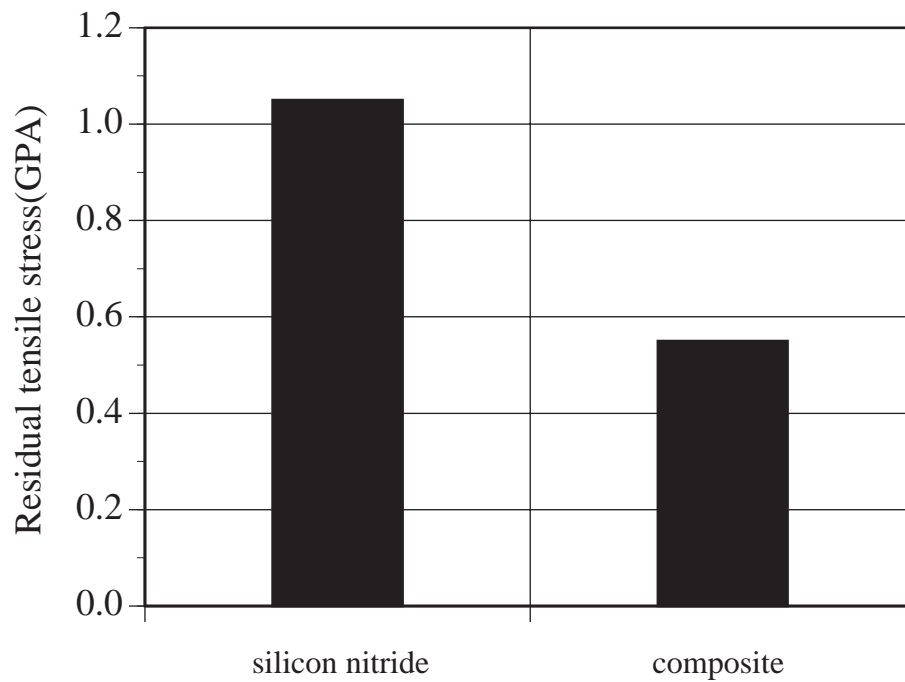


Figure 4.1 : Residual stress measurement of deposited films using substrate curvature method. A composite film consists of silicon nitride (2000 Å) and silicon dioxide (1500 Å).

Instead of making composite films, buckling of diaphragms due to compressive stress could be avoided by adjusting the dimensions of the diaphragms, such as the lateral dimensions or thickness. For given dimensions of a diaphragm, there exists a maximum compressive force which can be applied to a diaphragm along the middle plane of the diaphragm without buckling. The maximum compressive force is called the critical compressive force. Assume that a clamped-edges square diaphragm with length of  $a$  is compressed in its middle plane by force  $N$  (per unit length) uniformly distributed along the edges. When

$N$  is increased to a critical value ( $N_{cr}$ ), the flat diaphragm becomes unstable and buckles. This critical compressive force  $N_{cr}$  is given by

$$N_{cr} = 5.33 \cdot \frac{\pi^2 \cdot D}{a^2}, \quad (4.3)$$

where  $D$  is the flexural rigidity of a diaphragm with thickness  $h$  [9]. The flexural rigidity  $D$  of a film stack is

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)},$$

where  $E$  is the Young's modulus of the diaphragm and  $\nu$  is poisson's ratio of the diaphragm. Equation (4.3) implies that a diaphragm under compressive stress, like a silicon dioxide diaphragm, would be flat if  $h^3/a^2$  of the diaphragm is large enough to make  $N_{cr}$  bigger than the residual compressive stress. This fact will be considered when the fabrication process for the Fabry-Perot cavity sensor is designed in chapter 5.

## 4.2 DEFLECTION OF MULTIPLE FILM STACKS

In this section, the deflection of a diaphragm consisting of multiple dielectric films is calculated when external pressure is applied to the diaphragm. The diaphragm could be formed by either bulk micromachining techniques or surface micromachining techniques. The boundary conditions of the diaphragm depend on the technique the diaphragm is built with. To have simpler boundary conditions, the diaphragm formed using the bulk micromachining technique has

been considered because the clamped boundary conditions along the edges could be assumed for the diaphragm. The deflection of the square composite diaphragm is modeled as the deflection of a rigidly clamped square plate which has equivalent Young's modulus and equivalent residual stress of the composite diaphragm. Suppose the length and thickness of a plate are  $a$  and  $h$ , respectively. As shown in the last section, the composite film stack has equivalent Young's modulus and equivalent residual stress, which are calculated from equation (4.1). The equivalent residual stress ( $\sigma$ ) is assumed to be uniformly distributed throughout the film stack and is positive for tensile stress and negative for compressive stress. The deflection of the film by external pressure ( $q$ ) is assumed to be comparable to the thickness of the film stack. This assumption is valid for Fabry-Perot cavity pressure sensors which are designed in such a way that the movable mirror travels fractions of an optical wavelength for a full range of pressure to be measured. Under the above conditions, the deflection ( $\omega$ ) of the composite film stack is obtained by solving the differential equation

$$\frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{q}{D} + \frac{T}{D} \left( \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial x^2} \right), \quad (4.4)$$

where  $D$  is flexural rigidity of a film stack and  $T$  is equal to  $\sigma \cdot h$ .

Taking the coordinate origin at the center of the film stack, the boundary conditions for all edges clamped are

$$\omega = \frac{\partial \omega}{\partial x} = 0 \text{ at } x = \pm a / 2,$$

$$\omega = \frac{\partial \omega}{\partial y} = 0 \text{ at } y = \pm a / 2 .$$

Since obtaining an exact solution satisfying the above boundary conditions is very difficult, an approximate solution can be calculated using a numerical method, the Marcuse method. The Marcuse method has been shown to be sufficiently accurate for practical purposes [10, 11] . As an alternative, a more simplified solution can be obtained by solving Equation (4.4) under certain conditions. Details of the procedure are shown in Appendix A.

With the above boundary conditions, the deflection of a diaphragm has been calculated by the Marcuse method. For convenience in numerical calculation, the residual stress ( $T$ ) of the film stack is represented in terms of  $T_e$ , where  $T_e = 4\pi^2 D / a^2$ , and  $D$  is the flexural rigidity of the diaphragm. The solutions to the deflection equations of diaphragms with residual stress of zero and  $6T_e$ , as examples, are

$$\begin{aligned} \omega = \frac{K}{37.96} \{ & 1 - 0.349 \sin(2.087\xi) \sinh(2.087\xi) - 0.101 \cos(2.087\xi) \cosh(2.087\xi) \} \\ & \times \{ 1 - 0.349 \sin(2.087\varphi) \sinh(2.087\varphi) - 0.101 \cos(2.087\varphi) \cosh(2.087\varphi) \} \end{aligned} \quad (4.5)$$

and

$$\begin{aligned} \omega = \frac{K}{191.23} \{ & 1 - 0.21 \cosh(2.716\xi) + 0.0009 \cosh(7.2\xi) \} \\ & \times \{ 1 - 0.21 \cosh(2.716\varphi) + 0.0009 \cosh(7.2\varphi) \} , \end{aligned} \quad (4.6)$$

where  $K = \frac{a^4 \cdot q}{16D}$ ,  $\xi = \frac{2x}{a}$  and  $\varphi = \frac{2y}{a}$  .

Figure 4.2. shows a plot of the normalized deflection for a fixed external pressure versus residual stress of the diaphragm. The Young's modulus and thickness of the diaphragm are assumed to be 160 GPa and 0.8  $\mu\text{m}$ , respectively. For a fixed external pressure the deflection of the diaphragm at center, where maximum deflection occurs, decreases as residual stress increases. This implies that the sensitivity of Fabry-Perot cavity sensors would be degraded by higher residual stress in the moving mirror.

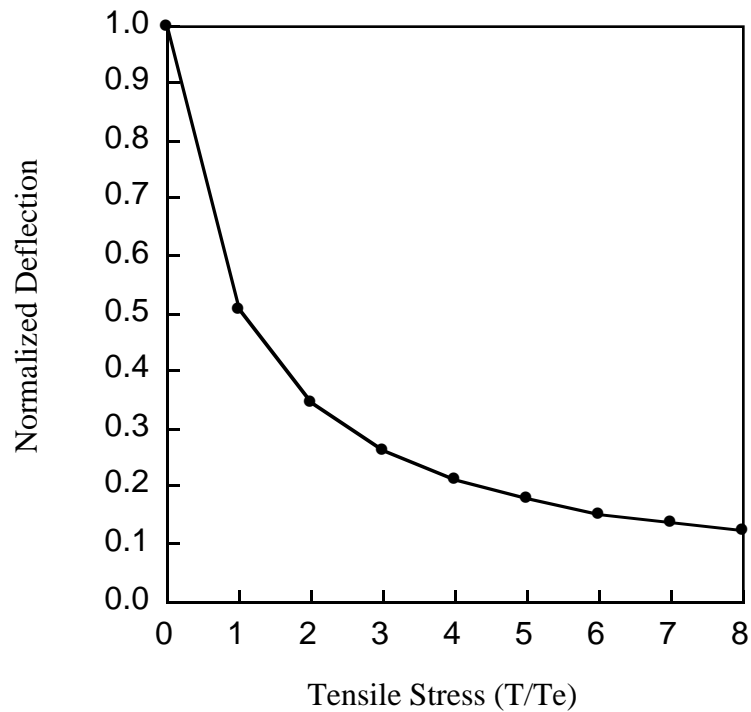
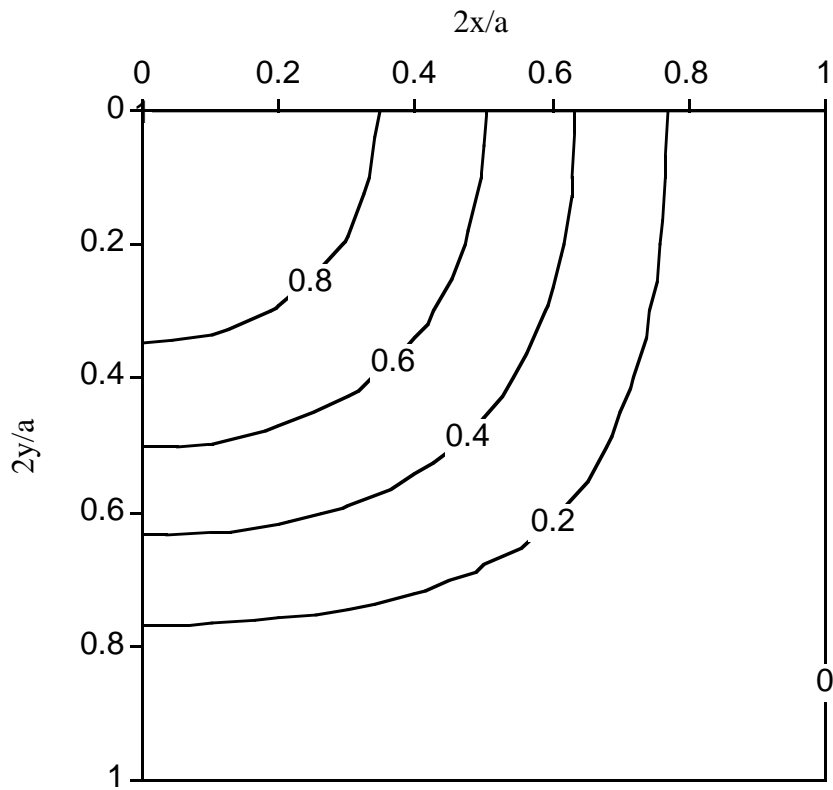


Figure 4.2 : Dependency of a diaphragm deflection at center on residual stress of the diaphragm.

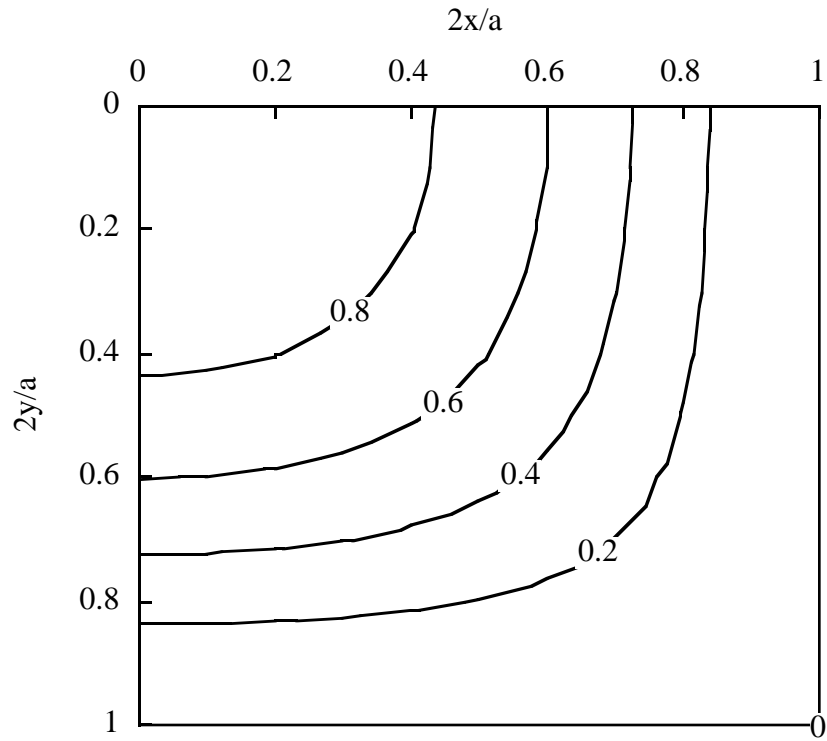




(a)

Figure 4.3 : Normalized deflection of diaphragm as a function of residual stress. (a) without residual stress; (b) with residual stress ( $6T_e$ ). Numbers on the contour lines represent normalized deflected height to deflection at center (origin in maps).

Also, the residual stress of the diaphragm makes the shape of the deflected film stack change. Figure 4.3 shows contour maps of deflected height generated using equation (4.5) and equation (4.6). From the maps, it is obvious that the deflected shape of a diaphragm with residual tensile stress becomes flatter as the residual tensile stress increases.



(b)

Figure 4.3 : continued.

### 4.3 DESIGN ISSUES FOR MECHANICAL COMPLIANCE

To match the compliance of a multiple film stack to the pressure range to be measured, there are several design variables, such as size, mechanical travel, and residual stress of the stack, to be considered. However, all design variables are not freely adjustable to achieve a desired compliance of the stack. For example, mechanical travel of the stack is usually upper-bounded by one half of

the wavelength of the illumination source to maintain a single-valued optical response. The mechanical travel can also be bounded by the optimum mechanical travel which is obtained as shown in chapter 3 when process-induced thickness variation is considered. For a fixed mechanical travel the compliance of the stack can be adjusted by changing the lateral size and residual stress of the stack. For example, for a simple geometry the compliance of the stack is proportional to the square of the area, as shown in equation (4.6). Tensile stress of the stack can decrease the compliance by up to a factor of 5 as shown in Figure 4.2. This residual stress can be controlled by changing the relative thickness ratio of the layers. For a Fabry-Perot cavity based pressure sensor, however, the tensile stress of the stack is not totally an independent variable since the thickness of each layer also affects the optical response of the cavity. Thus, adjusting the lateral size of the stack will be the easiest way to tailor the compliance of the stack for this type of pressure sensor.

#### **4.4 SUMMARY**

Mechanical properties of a multiple film stack, consisting of silicon dioxide and silicon nitride, were studied. Using beam curvature method the residual stresses of dielectric layers prepared by LPCVD were measured. In addition to residual stress, equivalent Young's Modulus of a multiple stack was calculated, assuming the films prepared in our facility have the same values of Young's Modulus as those in [5, 8] . The mechanical properties determine the compliance as well as the deflected shape of a multiple stack, which will be a moving mirror of a Fabry-Perot cavity based sensor. The deflection of a multiple stack was calculated using a numerical method, i.e. the Marcuse method, revealing the impact of residual stress on the compliance of a multiple stack. The compliance as well as the deflected shape of the multiple stack will be used to simulate the optical response of the Fabry-Perot cavity sensor.