Chapter 2

Analysis

2.1 Introduction

This chapter describes the methods of analysis used to calculate the various quantities that are needed in designing antennas and the dielectric stacks shown in Figure 2.1. Because of the complexity of the problem, the analysis is broken into two major parts. The first is the "radiation analysis" which allows the calculation of beam patterns, and the relative amounts of power delivered to air and to surface waves in the dielectric. The effects of dielectric losses and ground plane losses can be assessed in this analysis. The results from these calculations will comprise the bulk of the presented calculations because most of the important parameters of both the antenna and the dielectrics can be determined from this relatively simple approach. The amount of power radiated to guided waves can be determined in this type of analysis. This is one of the main characteristics that is needed to properly design the dielectric layers and the spacing between the elements in order to maximize the power radiated to air and minimize the power radiated to guided waves. The second step in the analysis is to calculate the input impedance and the field distributions in the slots. This technique combines the use of the so-called "spectral domain" method in conjunction with an application of the reciprocity theorem to calculate the coupling between a microstrip feed line and the slot antennas. In the case of the single slot fed by a microstrip line, the discontinuity of the slot can be modelled as a a lumped series impedance in the microstrip line. In the twin slot case,

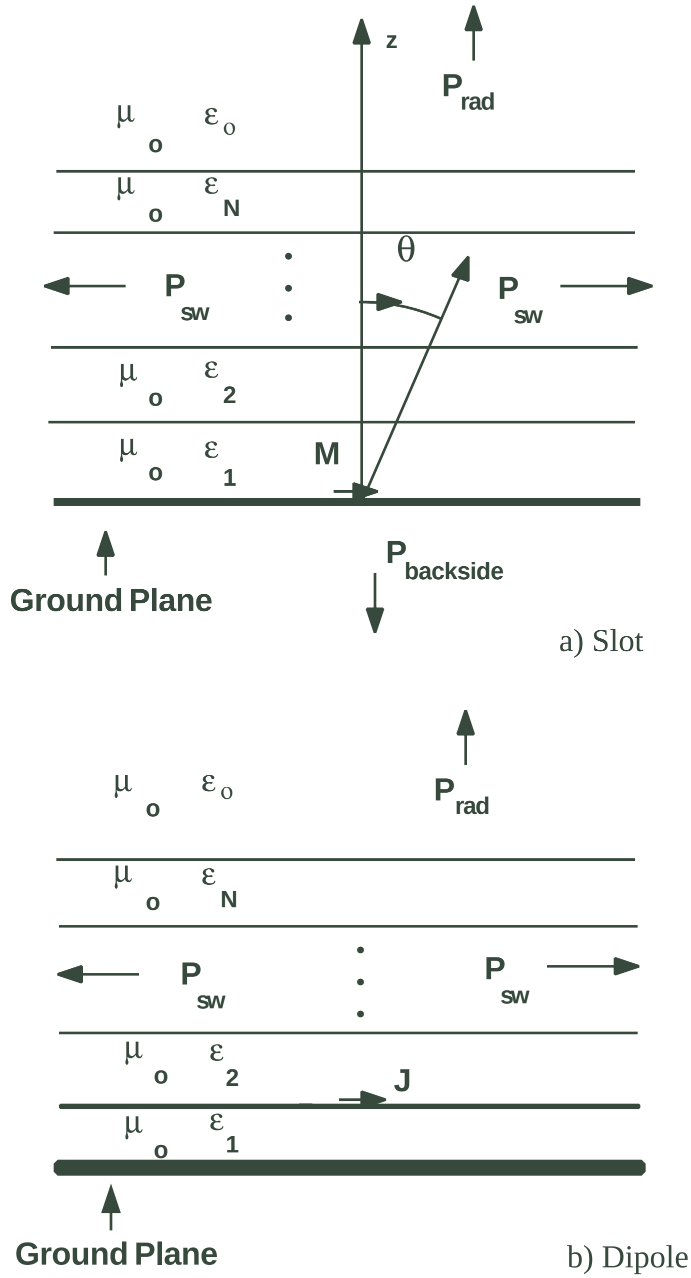

Figure 2.1 Slot (a) and dipole (b) antenna structures. All layers are one quarter of a dielectric wavelength thick. Layer N is adjacent to air. The ground plane is in the x-y plane.

the detector is centered between the slots and the feed line is terminated at the slot by a short circuit. In both of these cases a narrow slot approximation is used, and quasi-TEM fields are assumed to exist on the microstrip line.

In this work, all quantities are in MKS units and the harmonic time dependence is $e^{-i\omega t}$ for the analysis. Maxwell's equations then have the form:

$$\text{curl}\,\vec{H} = -i\omega\vec{D} + \vec{J}$$

$$\text{curl}\,\vec{E} = i\omega\vec{B} - \vec{M}$$

$$\text{div}\,\vec{B} = 0$$

$$\text{div}\,\vec{D} = \rho$$

Where $\vec{H}$ is the magnetic field, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic flux density and is related to $\vec{H}$ by $\vec{B} = \mu_0\vec{H}$, $\vec{D}$ is the electric flux displacement and is related to $\vec{E}$ by $\vec{D} = \varepsilon\vec{E}$, ρ is the free charge density in the medium, $\vec{M}$ is the magnetic current density, $\vec{J}$ is the electric current density, and ε and μ_0 are the scalar permittivity and permeability respectively of the medium under consideration. In this work we only consider non-magnetic materials, so μ_0 is always the permeability of free space, $4\pi x10^{-7}$ Henrys/meter.

2.2 Radiation Analysis

The term "radiation analysis" is used because the purpose of this analysis is to determine the radiated or far-fields produced by the source. This includes both the power radiated to air and the power coupled to guided modes. This analysis is useful because the radiated or "far" fields are only weakly dependent on the exact details of the current. Thus we can use relatively simple models for the assumed source currents and obtain accurate calculations of the radiated fields. For convenience, the calculations of radiated power are broken into two parts: 1) radiation-to-air, and 2) guided (surface) waves. The radiation-to-air model is recursive and includes both dielectric and ground-plane losses. The guided wave calculations are also recursive and can accommodate an arbitrary number of layers. The power coupled to the guided waves is calculated by a reciprocity method described in [1] by Rutledge. This method is simple, gives the same information as the much more tedious task of evaluating the fields from the poles of a Sommerfeld-type integral, and has the additional advantage of providing some physical insight into the coupling between the guided waves and the source. Although the calculations of power delivered to the guided waves do not include material losses, this should not be a significant limitation for the low loss materials discussed.

Radiation-to-Air Model

There are several approaches which have been used in the past to calculate the fields radiated-to-air. To calculate the far-field pattern of a dipole imbedded in a layered substrate perhaps the most straightforward approach is to use the reciprocity theorem and a transmission line model [15,16]. This approach is also applicable to slot antennas on layered substrates. Another approach is to calculate the radiated fields from a Sommerfeld-like integral using the method of stationary phase (saddle point approximation), although this can become quite cumbersome for multi-layered structures [15]. A simplified version of this approach is given in Appendix A. The method used here is to first find the plane wave angular spectrum generated by the antenna, and then associate with each of these spectral components a transmission line model for the dielectric stack [33], as shown in Figure 2.2. For each component, the antenna is now represented as a circuit source, using a current source for dipoles, and a voltage source (or equivalently, a magnetic current) for slots [1]. One advantage of this approach is that we have an equivalent circuit model which is simple to interpret. Also, we can use this same approach when we want the "complete" plane wave spectrum when we need the Green's function for the impedance calculations.

Analysis of the slot and dipole are very similar so we will show only the analysis for the slot. For an elemental slot in a perfectly conducting ground plane, we use Huygen's equivalence principle [34], and replace the aperture with an equivalent magnetic current:

$$\vec{M} = \vec{y}\, \delta(x)\, \delta(y)\, \delta(z) \tag{2.1}$$

This element of current can be decomposed into harmonic current sheets in the x-y plane. Each sheet can then be further decomposed into components which give rise to TE (to the z-axis) and TM (to the z-axis) fields. The resulting expressions for the slot are:

$$\vec{m}^{TE} = (\vec{x} \cos\phi + \vec{y} \sin\phi\,) \sin\phi\; e^{ik_x x\, +\, ik_y y} \tag{2.2}$$

$$\vec{m}^{TM} = (-\vec{x} \sin\phi + \vec{y} \cos\phi) \cos\phi\; e^{ik_x x\, +\, ik_y y} \tag{2.3}$$

$$k_x = k_0 \sin\theta \cos\phi$$

$$k_y = k_0 \sin\theta \sin\phi$$

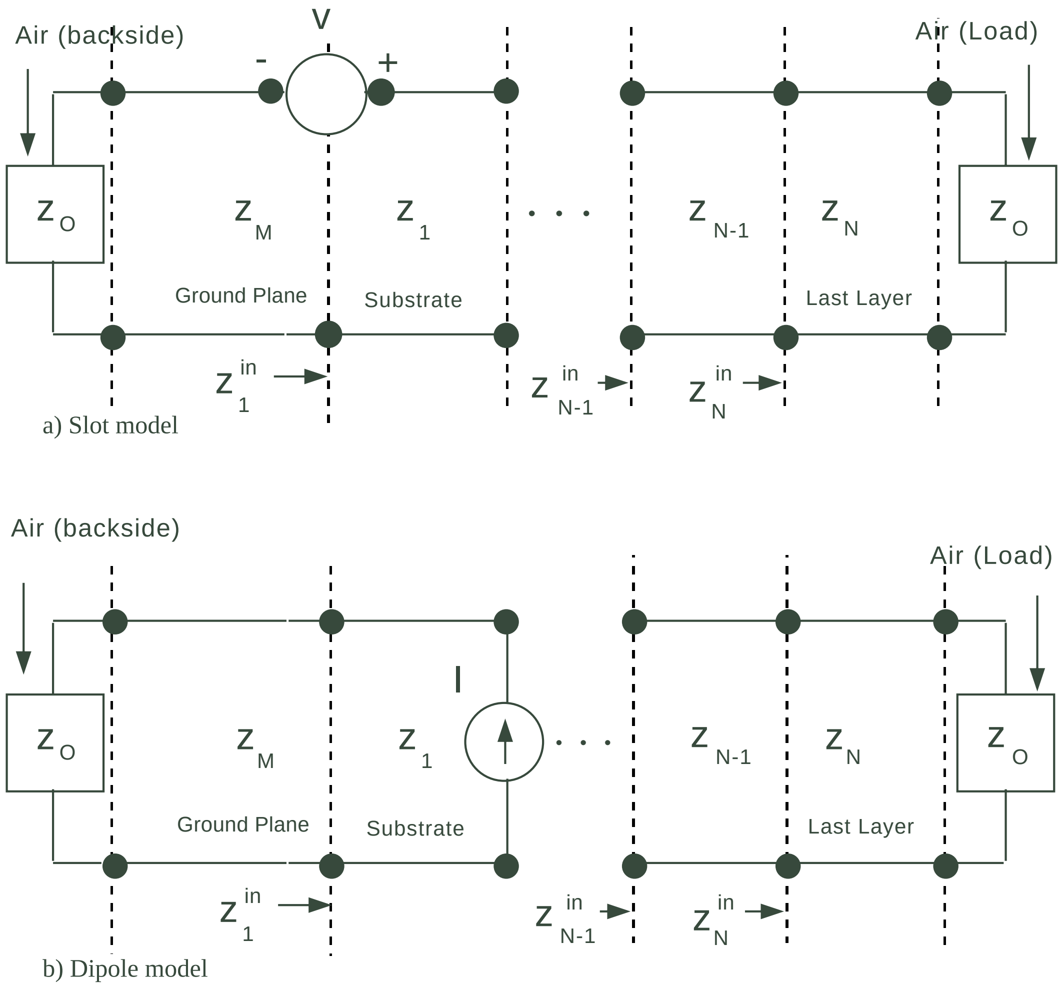

Figure 2.2 Transmission line models used to calculate the radiated-to-air fields after the TE-to-z and TM-to-z decomposition. Note that the ground plane is modelled as a finite length stub of transmission line: (a) slot, (b) dipole.

where $\vec{m}^{TE}$ is the component of the y-directed magnetic current that excites TE waves, $\vec{m}^{TM}$ is the component of the y-directed current that excites TM waves, ϕ is the polar angle of the k-vector in the x-y plane, θ is the angle from the z-axis in free space, and k_o is the wavenumber in free space. The voltage source V in Figure 2.2 is just the magnitude of $\vec{m}$. For an electric current source (i.e. the dipole), we would interchange the expressions for TE and TM currents; otherwise the expressions for the decomposed current are unchanged.

After the decomposition into TE and TM components, the behavior of a stack of dielectrics for each angle of incidence can be lumped into a single surface impedance looking into the n^{th} layer, Z_n^{in}, using the recursive formula [35]:

$$Z_n^{in} = \frac{Z_{n+1}^{in} \cos(\psi_n) - i\, Z_n \sin(\psi_n)}{Z_n \cos(\psi_n) - i\, Z_{n+1}^{in} \sin(\psi_n)}\, Z_n \qquad (2.4)$$

where Z_{n+1}^{in} is the surface impedance looking into the n+1 layer, numbered as shown in Figure 2.2, and Z_n is the appropriate TE or TM wave impedance of the n^{th} layer. The term ψ_n represents the electrical length of the n^{th} layer, and is given by $\psi_n = k_n \cdot \gamma_n \cdot d_n$, where $k_n = \omega \cdot \sqrt{\mu_o \varepsilon_o \varepsilon_n}$, ε_n is the relative dielectric constant of the n^{th} layer, and d_n is the thickness of the n^{th} layer. The transverse cosine γ_n is found using the continuity boundary conditions for the parallel components of E and H at the interfaces between the dielectrics, which demands that the parallel components of the k-vectors must be the same throughout the layers. From this condition, γ_n in each layer can be found from:

$$\gamma_n = \sqrt{1 - \frac{k_\rho^2}{k_n^2}} \tag{2.5}$$

where $k_\rho^2 = k_x^2 + k_y^2$. The TE and TM wave impedances are now just $Z_n^{TM} = \gamma_n \cdot \sqrt{\frac{\mu}{\varepsilon_n}}$ and $Z_n^{TE} = \frac{1}{\gamma_n} \cdot \sqrt{\frac{\mu}{\varepsilon_n}}$. These definitions are applicable for complex values of ε_n and γ_n. Consequently material losses can be included in a straightforward manner. This method of calculating the refracted fields though a lossy dielectric is more straightforward than trying to compute the complex angles of refraction. The difficulty of dealing with complex angles is avoided because the problem is described in terms of γ and k_ρ. Imbedded in equations (2.4) and (2.5) is the complete information about the electromagnetic behavior of the dielectric stack as a function of k_ρ. It is this simplification of the behavior of the stack to a single number for a given value of k_ρ that makes the analysis of the radiators on layered substrates straightforward.

This same treatment can also be used for the ground plane by treating it as a stub of transmission line with a large conductivity (Figure 2.2). It is important to be able to include the finite conductivity and thickness of the ground plane, since in some cases the surface impedance produced by the dielectric stack is comparable to the surface impedance of the ground plane. Typical thicknesses of metal deposited by evaporation are from about .01µm to about 1µm, and the skin depth of copper at 94GHz is about .24µm, so the finite thickness of the metal should be taken into account. In many cases, however, it is reasonable to approximate the ground plane as a perfect conductor, hence the stub of transmission line can be replaced by an ideal short.

When finite ground plane conductivity is included, the model for the slot is approximate. The approximation occurs when Huygen's principle is used to replace the E-field in the aperture with an equivalent magnetic current. The requires the assumption that the E-field tangential to the ground plane is zero everywhere outside of the aperture. This is only true if the ground plane is a perfect conductor. If the ground plane has a high conductivity, the tangential E-field should be small compared to the E-field in the aperture. This approximation should yield accurate results for our radiation-to-air calculations. The transmission line model for the dipole, however, is exact since the spectral components of electric currents in the model (Figure 2.2 b) are the components of the actual electric currents flowing in the dipole and not equivalent electric currents.

Using the recursive method discussed above, we can calculate the fields at the boundary between free space and the last dielectric layer (layer N) due to the original elemental source. We can then use the equivalence principle in combination with the method of images to reduce the problem to the calculation of the far-field pattern of a current (or field) distribution over a conducting ground plane. The integral given by Eq. 3-17 in [34] is :

$$\vec{E}(\vec{r}) = -\text{curl} \iint_{\text{x-y plane}} \frac{e^{ik_o|\vec{r} - \vec{r}'|}}{2\pi\, |\vec{r} - \vec{r}'|}\, \vec{E}_t(\vec{r}) \times d\vec{S}$$

which can be approximated in the far-field as:

$$\vec{E}(r) = \frac{e^{ikr}}{2\pi r} (\vec{k} \times \vec{E}_t (k_x, k_y) \times \vec{z}) \tag{2.6}$$

where $\vec{E}_t$ is the tangential field at the interface, calculated from the transmission line model and plane wave spectrum of the original source. This is the same result as that obtained by reciprocity [15] or by evaluating the inverse transform integral in the far-field by the method of stationary phase as done in Appendix A.

Guided Waves

The power delivered by the antenna element to guided modes in the dielectric stack can be obtained by using the reciprocity theorem. This technique is discussed in [1] with regard to a single layer substrate. The technique is extended here to cover N lossless layers. The fields considered are of the form of simple two-dimensional guided waves in the slab propagating in one direction. The reciprocity test field is a single guided wave propagating from one side of the antenna to the other. The source field is an outward traveling wave with unknown amplitude and angular dependence, with the same functional z-dependence as the test field. For TE waves the only E-field component is in the y direction, so the expression for the n^{th} layer is:

$$E_y = \Lambda_n \left(e^{ik_n\gamma_n (z_n - d_n)} + R_n\, e^{-ik_n\gamma_n(z_n - d_n)}\right) e^{ik_n\beta_n x} \tag{2.7}$$

where Λ_n is the unknown amplitude to be found using reciprocity, R_n is the TE E-field Fresnel reflection coefficient, k_n, γ_n, and d_n are defined as before, z_n is a relative z-coordinate, which is zero at the left-hand boundary and d_n at the right hand boundary of the layer (as shown in Figure 2.2), and $\beta_n^2 = 1 - \gamma_n^2$. The form of the fields is the same as for the radiation-to-air case, except that for surface waves the transverse cosine γ in air is purely imaginary. Writing the waves due to the source in the form of two dimensional waves in the slab implicitly assumes that the observation point is far away from the source in the x-y plane.

In order to make use of equation 2.7, for a particular propagating guided mode, the values of γ_n in each layer of the dielectric stack must be found. The approach of using different functional forms for the many different cases becomes cumbersome when more than two or three slabs are considered. To solve this problem, the method of transverse resonance, in combination with equation 2.4, can be manipulated to give an eigenvalue equation for the values of γ_n. For the case of a grounded substrate γ_1 is found from:

$$0 = A_2\, Z_2 \cos(\psi_1) - i\, B_2\, Z_1 \sin(\psi_1)$$

where $\psi_1 = k_1 \cdot \gamma_1 \cdot d_1$, and A_2 and B_2 are found using the recursive relations:

$$A_n = A_{n+1}\, Z_{n+1} \cos(\psi_n) - i\, B_{n+1}\, Z_n \sin(\psi_n)$$

$$B_n = B_{n+1}\, Z_n \cos(\psi_n) - i\, A_{n+1}\, Z_{n+1} \sin(\psi_n)$$

and for the N^{th} (last) layer:

$$A_N = Z_0 \cos(\psi_N) - i\, Z_N \sin(\psi_N)$$

$$B_N = Z_N \cos(\psi_N) - i\, Z_0 \sin(\psi_N) \qquad (2.8)$$

where $\psi_n = k_n \cdot \gamma_n \cdot d_n$. At a particular frequency, the modes that are near cutoff will tend to have real transverse cosines γ_n in all of the slabs, while the lower order modes that are well above cutoff will have imaginary γ_n in the lower dielectric constant slabs. Consequently, modes well above cutoff will tend to be confined to the layers with the higher dielectric constants. The values of k_ρ which correspond to the propagating modes are the values of k_ρ at which the Green's function for the slot (discussed in the next section) has simple poles.

The radiation efficiency of antennas on these layered structures can now be determined. We will define the efficiency for slots using:

$$\eta_{slot} = \frac{P_{rad}}{P_{rad} + P_{sw} + P_{backside}} \tag{2.9}$$

where P_{rad} is the total integrated power radiated to air through the dielectric layers, P_{sw} is the total power coupled to the guided (surface) waves, and $P_{backside}$ is the total power radiated to the backside (directly to air) of the ground plane. The efficiency of the dipole is defined similarly using:

$$\eta_{dipole} = \frac{P_{rad}}{P_{rad} + P_{sw}} \tag{2.10}$$

Here there is no backside radiated power because the dipole is over the ground plane, so P_{rad} is the total power radiated to the air through the dielectric layers above the element. Note that since the defined efficiencies are based on total integrated power, they do not give information about the antenna pattern. Thus, two structures with the same efficiencies may not be equally useful, depending on the actual details of the patterns (equation 2.6).

2.3 Reciprocity and Spectral Domain Analysis

An antenna structure is usually coupled to a feed line which is, in turn, connected to some sort of receiver or transmitter. It is usually desirable to have maximum power transfer between the antenna-feed structure and reciever or transmitter circuit. Therefore, a very important parameter to be able to determine is the input impedance looking into the antenna and associated feed line. The structure we are interested in analyzing is shown in Figure 2.3. The detector, the point at which we wish to know the impedance, is located between the two slots as shown. It is coupled to each of them by a microstrip feed line, and what we need to know is the impedance seen by the detector looking into the microstrip circuit. Calculating the input impedance or the near fields of the antenna requires calculation of the current distribution on the antenna. If the antenna has dimensions that are the order of a wavelength, the currents can be found by using a technique called the method of moments (MOM) [36]. The method of moments is a technique which converts an integral equation into a system of linear matrix equations. The unknowns in this problem are the fields in the slots and the MOM technique approximates this solution as a weighted sum of known functions called basis functions. The resulting matrix equations are solved to determine the unknown weighting coefficients. Since this approach assumes that the unknown currents can be well approximated by a weighted sum of these basis functions, a careful choice of basis functions is needed. It is also necessary to assume that the ground plane conaining the slot is a perfect conductor so the E-fields tangential to the ground plane will be zero everywhere except in the slots.

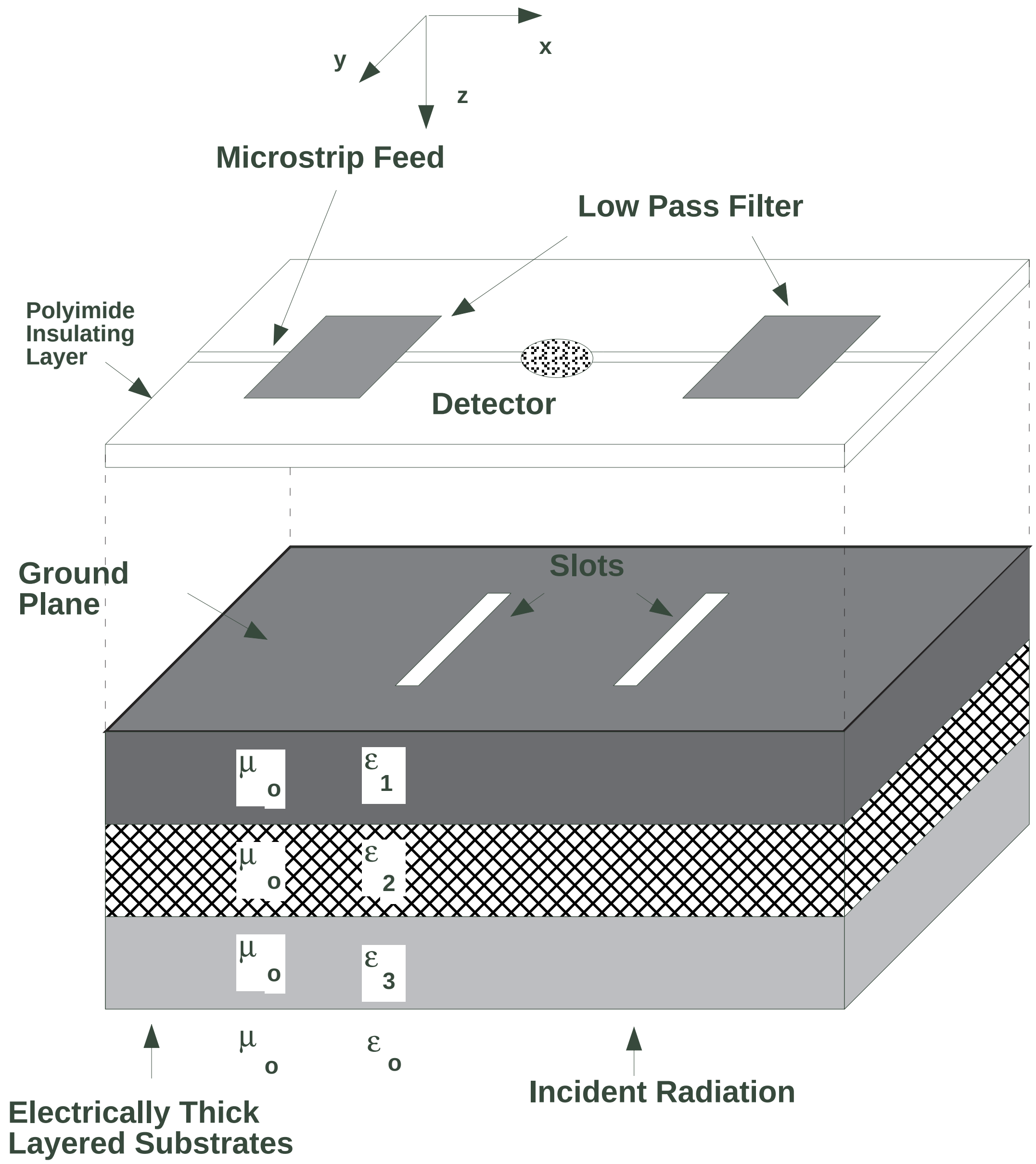

Figure 2.3 Diagram of twin slot antennas fed by a microstrip line with the detector centered between the two antennas. The dielectric supporting the microstrip line is very thin compared to a wavelength while the supporting substrate, layer 1, is electrically thick.

The type of MOM calculation used here requires knowledge of the spatial behavior of the Green's function appropriate for the problem. This is not known in closed form for problems involving sources over, or imbedded in, layers of dielectrics. However, if the layers are uniform the x-y plane, then the Green's function can be determined by breaking the source down into its plane-wave components (harmonic current sheets in the x-y plane) and determining the fields for each component from the appropriate transmission-line model associated with each spatial frequency (similar to the procedure in the previous section). The spatial Green's function is constructed by summing over all the plane waves generated by the source. Once this is done, the MOM technique can be applied in a straightforward manner as is done by Rana and Alexopoulos [37]. It is, however, very computationally intensive to calculate the Green's function in this manner for every point of the integral. Another approach would be to examine the integral matrix elements formulated after enforcing the boundary conditions in the space domain and realize that if all of the unknown currents lie in a infinitesimally thin plane, the integral can be written the in k_x-k_y domain [24,38].

The closed form for the Green's function in the k_x-k_y domain can be found in this case, thus a closed form expression for the integrand can be obtained. This is the spectral domain approach that we will use to calculate the fields in the aperture of the slots. The Green's function contains all of the information about the layers of the dielectrics. The particular Green's function needed depends on the nature of the problem to be solved and can be determined in a straightforward manner from the method described in Appendix B.

Coupling to the Feed Line by Reciprocity

In the problem we are trying to solve we also need to calculate the impedance seen by an electrically small detector mounted in a microstrip feed line that is electromagnetically coupled to the slot antenna. There have been several ways of approaching this problem reported in the literature. The method that we use here is similar in principle to D.M. Pozar's reciprocity method for calculating the currents produced on a microstrip line by the slots [39]. This method assumes that the fields on the microstrip line are quasi-TEM, hence the fields on the line are simple to work with. This method does not enforce any boundary conditions on the microstrip feed line so the scattering off of the metal feed line is ignored. These approximations should be good when the slot is narrow and the microstrip line is close to the slot and narrow compared to the width of the slot antenna. We will begin by using the reciprocity theorem to give a relationship between the currents in the microstrip line and the fields in the slot antenna. We will then enforce continuity of the H-fields in the slots and employ the spectral domain MOM to calculate the fields in the slots.

In this section we show the reciprocity calculations for a microstrip-fed twin slot antenna that is operated in the even mode to reduce the power delivered to guided waves in the substrate. A representation of the circuit problem we are trying to solve is shown in Figure 2.4. The impedance that we wish to know is at the plane containing $\vec{M}^A$ in Figure 2.4 and can be found from:

$$Z_{in} = \frac{V}{I_{imp} + I_{sc}} \tag{2.11}$$

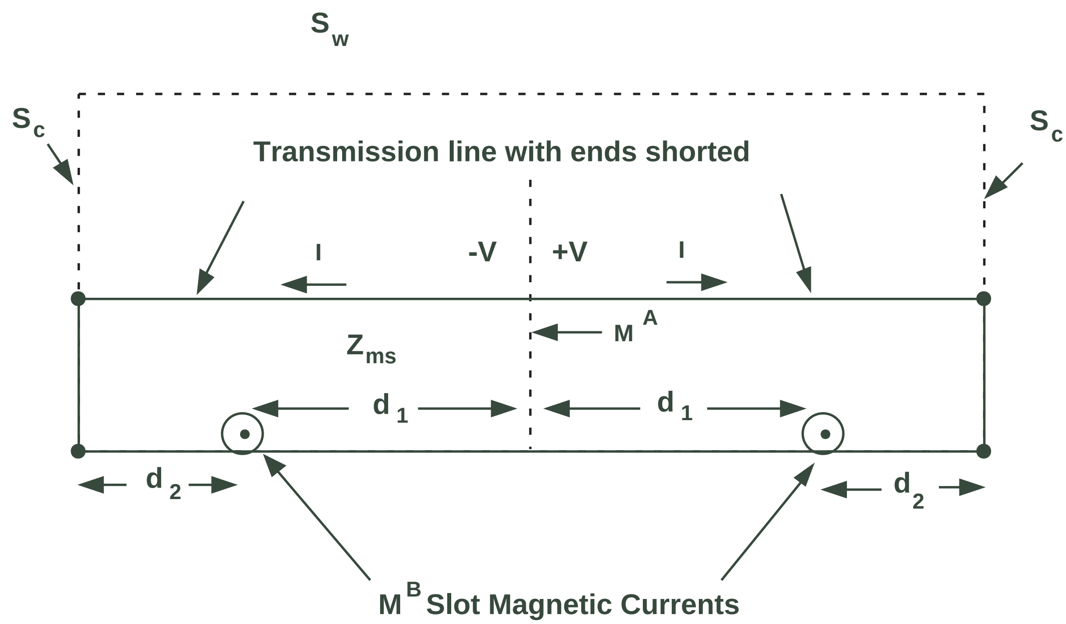

Figure 2.4 Transmission line model of the microstrip feed network where the slots are modelled as the magnetic currents, M^B, over a closed ground plane. The ends of the lines are shorted, the distance from the slot to the short is d_2, the distance from the slot to the detector (center vertical dotted line) is d_1, the characteristic impedance of the transmission line is Z_{ms}, a voltage source is placed at the detector point with a discontinuity of 2V and is modelled as the magnetic current sheet M^A. S_c is the cross sectional surface of the line, S_w is the surface that is the enclosing "wall" of the microstrip line, and I is the total microstrip current $I_{imp}+I_{sc}$.

where I_{imp} is the current due to the voltage source shown as $\vec{M}^A$ in Figure 2.4. This is just the current on the microstrip line without the slot discontinuitues. The 'scattered' current, I_{sc}, is the current on the microstrip line that is induced by the magnetic currents in the slots. The reciprocity analysis assumes the fields due to the currents in the microstrip line are quasi-TEM, and can be written as:

$$\vec{h} = h_y \vec{y} + h_z \vec{z}$$

$$\vec{e} = e_y \vec{y} + e_z \vec{z}$$

where $\vec{e}$ and $\vec{h}$ are the quasi-TEM E-field and H-field respectively. The ends of the microstrip feeds are terminated in ideal short circuits. Thus the fringing fields due to the terminations of the strip are neglected. The structure is assumed to be symmetric about the x-z plane where the detector is located, and the voltage source across the gap is modelled as a simple discontinuity in the E-field on the microstrip line as:

$$\vec{M}^A = 2\, \vec{e} \times \vec{x} \tag{2.12}$$

where $\vec{M}^A$ is the source magnetic current sheet in the plane (y-z plane) perpendicular to the microstrip feedline The reciprocity theorem can be applied as shown in Figure 2.4. The slots are modelled as magnetic current line sources and are assumed to be equal in amplitude. This is an even mode excitation since the electric fields are pointed in the same direction. We are not interested in the odd mode here because it radiates to air poorly and would not make a good mode of operation for an antenna. The integrals over the enclosing "walls" of the microstrip line (see Figure 2.4), S_w, vanish because the surface normal is orthogonal to the $\vec{e} \times \vec{h}$ product. The surface integrals of the fields over the cross-sectional surface, S_c, are chosen to be at the short circuits where the E-fields are zero so they vanish as well. The reciprocity theorem reduces to:

$$\int \vec{H}^A \bullet \vec{M}^B dV = \int \vec{H}^B \bullet \vec{M}^A dV \tag{2.13}$$

where the "A" fields are the fields due the voltage source, and the "B" fields are due to the fields in the slots.

The quasi-TEM fields are normalized so that:

$$\int_{S_a} (\vec{e} \times \vec{h}) \bullet d\vec{S} = 1 \tag{2.14}$$

where $\vec{e}$ and $\vec{h}$ only have components in the y and z directions. The currents on the transmission line are assumed to have sinusoidal dependence in the x-direction because of the closed ends of the lines.

Currents due to the impressed voltage source are easily determined.by:

$$I_{imp} = \frac{V}{2\, i\, Z_c \tan(\xi)} \tag{2.15}$$

where ξ is $k_m(d_1+d_2)$, k_m is the microstrip wavenumber, and $V=2\sqrt{Z_c}$, and Z_c is the characteristic impedance of the microstrip line. The current and hence the H-field, can be assumed to be a standing wave on the line in the x-direction with a maximum at the short circuit. In this situation, the current on the microstrip line will have the cosine dependence of: $I_{imp}(x) = A\cos(k_m(x-d_1+d_2))$. The amplitude A is determined by equation 2.15. The resulting expression for the transverse field on the microstrip line is:

$$\vec{H}^A(x) = \vec{h}\, i \cot(\xi) \cos(k_m(x-d_1+d_2)) \tag{2.16}$$

for the x-dependence of the H-field.

The currents due to the slots, I_{sc}, can be written as having a piecewise continuous sinusoidal dependence on x since we expect standing waves on the closed line. If we consider each source independently we can find the currents due to each individual slot and then sum up the contributions from each slot, which should be the same at the center due to the symmetry of the problem. From the boundary conditions, the current due to the slot on the right in Figure 2.4 has the following dependence:

$$I_{sc}^{1}(x) = A\cos(k_m(x - d_1)) \tag{2.17a}$$

$$I_{sc}^{2}(x) = B\cos(k_m(x + d_1 + d_2)) \tag{2.17b}$$

Where I_{sc}^{1} is the current to the right of the slot antenna and I_{sc}^{2} is the current to the left of the slot antenna. The continuity of the current in the strip across the slot establishes the following relationship between A and B:

$$B = \frac{\cos(k_m d_1)}{\cos(k_m(2d_2 + d_1))} A \tag{2.18}$$

To find the expressions used in the reciprocity integral we will begin by dealing with the left hand side of equation (2.13). This can be written as:

$$\int_{S_{slot}} ds\, H_y^{A}(x,y)\, M_y^{B}(x,y) = \int_{Strip} ds' \int_{Slot} ds\, J_x(x',y')\, G_{yx}^{HJ}(x,y\,;x',y')\, M_y^{B}(x,y) \tag{2.19}$$

where ds refers to the integral over the unprimed x-y coordinates and ds' refers to the integral over the primed x-y coordinates. The integral on the right hand side of equation (2.13) can be rewritten in the spectral domain by performing a Fourier transform in the x-y plane. The magnetic current M_y^{B} can be well approximated by a weighted sum of piecewise sinusoidal basis functions in the y-direction and either uniform, or the "singularity" basis function in the x-direction, both of which can be found in Appendix C [39]. For narrow slots, the Fourier transform in the x-coordinate of the slot current M_y can be approximated as 1. If one basis function is used we have:

$$\int \vec{M}^B \bullet \vec{H}^A \, dV = \frac{a}{2\pi \sqrt{Z_c}} \int_{-\infty}^{\infty} dk_y \, J_x(k_y) \, G_{YX}^{HJ}(k_m, k_y) \, M_y^B(k_y)$$

$$\int \vec{M}^B \bullet \vec{H}^A \, dV = a \, \Delta v \tag{2.20}$$

where k_m is the microstrip propagation constant (as before) and Z_c is the characteristic impedance of the microstrip line. J_x is the Fourier transform on the microstrip current in the y-direction, M_y^B is the Fourier transform of the slot magnetic current basis function in the y-direction and a is the unknown amplitude of the magnetic current in the slot. The factor of $1/\sqrt{Z_c}$ is the normalization factor which is the amplitude of the current that is required to create an H-field of $\vec{h}$ on the microstrip line. This can be obtained from power considerations by demanding VI = 1, and then using equation (2.13). If multiple basis functions are used, the integral can be written as the weighted sum of integrals:

$$\int dV \, \vec{M}^B \bullet \vec{H}^A = \sum_{\iota=1}^{N} a_i \, \Delta v_i \tag{2.21}$$

where the a_i 's are the weighting factors and Δv_i is the integral in equation (2.20) over the ith piecewise sinusoidal basis function $m_y{}^i$.

The left hand side of this integral is somewhat easier to deal with. The integral can be expressed as:

$$\int dV\, \vec{M}^A \bullet \vec{H}^B = 2\, A \int dy\, dz\, \vec{h} \bullet (2\, \vec{e} \times \vec{x}) \qquad (2.22)$$

The term under the integral can be re-written using $\vec{h} \bullet (2\vec{e} \times \vec{x}) = -\vec{x} \bullet (\vec{e} \times \vec{h})$, and the integral on the right-hand side is simply 2. The result from equation 2.14 is:

$$\int dV\, \vec{M}^A \bullet \vec{H}^B = -4A \qquad (2.23)$$

where A is the amplitude of the normalized H-field on the microstrip line due one of the two slots.

Spectral Domain Analysis

We now have one equation relating the amplitude of the normalized quasi-TEM fields on the microstrip line to the amplitudes of the magnetic currents in the slots. The remaining equation to solve the problem comes from enforcing continuity of the y-component of the H-fields across the slot aperture:

$$H_y^f + H_y^{m_b} = H_y^{m_s} \qquad (2.24)$$

where H_y^f is the H-field due to the total feed line currents, $H_y^{m_b}$ is the H-field due to the magnetic currents on the backside (feed line side) of the ground plane, and $H_y^{m_s}$ is the y-directed H-field due to magnetic currents on the substrate side of the ground plane. The H-fields due to the total feed current can be written as:

$$\vec{H}^f = \vec{h}\ \left(i \cot(\phi) \cos(k_m d_1) + A\left(\frac{\cos(k_m d_1)^2}{\cos(k_m d_2)} + \cos(k_m d_1) \right) \right) \qquad (2.25)$$

where h_y is the y-component of the fields due to the microstrip line with a current of $1/\sqrt{Z_c}$. If the left hand side of equation 2.25 is examined, we see that the first term in the brackets on the left-hand side is the field due to the impressed current, and the two terms inside the inner brackets multiplied by A are due to the currents induced by the slots.

There are a number of ways that the boundary conditions can be enforced. In our case we will enforce continuity of the H_y field across the slot in the integral sense using the basis functions themselves as weights. This amounts to evaluating the H_y fields in the slot due to all of the sources, then multiplying by the basis functions and integrating the resultant product over the slot aperture. The resulting system of equations can be solved for the unknown weighting coefficients of the basis functions, and we have solved the problem approximately. This amounts to rewriting equation 2.23 as:

$$H_y^f = H_y^{m_s} - H_y^{m_b} \tag{2.26}$$

and the fields can be written in terms of the currents as:

$$\int J_x(y') e^{i\beta x} G_{yx}^{HJ}(x, y ;x' ,y') \, dx \, dy = \int \sum_{i=1}^{N} a_i m_y^i(x' , y') G_{yy}^{HM}(x, y ; x', y') \, dx \, dy \tag{2.27}$$

where G_{yy}^{HM} is defined so that the integral over the magnetic currents in the slot is the difference of the H_y fields on the left hand side of equation 2.26, the coefficients a_i are the same as defined in equation 2.21, and the basis functions $m_y{}^i$ are the basis functions used in equation 2.21 (piecewise sinusoidal in y and uniform in x for the slot). This method of using basis functions as the weighting functions to enforce the boundary conditions is called Galerkin's method [36].

The required Green's function for one side of the ground plane derived in

Appendix B is:

$$G_{yy}^{HM} = \frac{\sin^2\phi}{Z_{TE}(k_\rho)} + \frac{\cos^2\phi}{Z_{TM}(k_\rho)} \tag{2.28}$$

where ϕ and k_ρ are the same as in the previous section. Z_{TE} is the TE surface impedance looking into the stack from the ground plane, and Z_{TM} is the TM surface impedance looking into the stack from the ground plane. The basis functions used are piecewise sinusoidal in the y-direction and uniform in the x-direction (reasonable for narrow slots [39]). The exact forms for the basis functions are given in Appendix C. If we interchange the summation and the integraion on the right hand side of equation 2.27, then generate N equations to solve for the N unkown weight coefficients of the basis functions. This is accomplished by multplying both sides of equation 2.27 by a basis function and integrating the resulting product. This is repeated until all of the basis functions have been used and we have N equations. The right hand side becomes the admittance matrix [Y]. The elements of both [Y] can then be written the the spectral domain form. The resulting integral in k_x and k_y for the ij admittance matrix element is :

$$Y_{ij} = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \, m_y^i (k_x,k_y) \, G_{yy}^{HM} (k_x,k_y) \, m_y^j (k_x,k_y) \tag{2.28}$$

in polar coordinates in the k_x-k_y plane this reduces to:

$$Y_{ij} = \int_0^\infty dk_\rho \, k_\rho \left(\frac{F_{TE}(k_\rho)}{Z_{TE}(k_\rho)} + \frac{F_{TM}(k_\rho)}{Z_{TM}(k_\rho)} \right) \tag{2.29}$$

where:

$$F_{TE}(k_\rho) = \int_0^{2\pi} d\phi\, F_p^2(k_y h)\, F_u^2(k_x w_s) \cos(k_x d_s) \cos(k_y h) \sin(\phi)^2 \qquad (2.30a)$$

$$F_{TM}(k_\rho) = \int_0^{2\pi} d\phi\, F_p^2(k_y h)\, F_u^2(k_x w_s) \cos(k_x d_s) \cos(k_y h) \cos(\phi)^2 \qquad (2.30b)$$

$$k_x = k_\rho \cos(\phi) \quad k_y = k_\rho \sin(\phi)$$

where w_s is the width of the slot antenna, h is the distance between the centers of the basis functions (and in the case of the piecewise-sinusoidal basis functions it is also the half-length of the function), d_s is the separation between the slots. and * denotes the complex conjugate. For the values of k_ρ between k_0 and k_d ($k_d = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_d}$) the integral has simple poles. These are evaluated in the Cauchy principal value sense [40] which means that the result is obtained by integrating numerically to within $0.01k_0$ of the pole and then adding the residue at the pole multiplied by $i\pi$. The numerical integrations were performed using Gaussian quadrature over the finite intervals for k_ρ less than k_0 and between the poles, but the aymptotic tails of the integral were integrated using Gauss-Laguerre interpolation. After multiplication, the right hand side of equation 2.27 will contian the column vector [Δv] the same as in equation 2.21. This results from "testing" the H-field in the slots due to the feedline currents. The ith tested feed line component of the H-field over the slot due to the currents in the feed line is:

$$\int_{Slot} dx\, dy\, H_y^f(x,y)\, m_y^i(x,y) = \Delta v_i \left(G + \left(\frac{\cos(k_m d_1)^2}{\cos(2 k_m d_2)} + \cos(k_m d_1)\right) A\right) \qquad (2.31)$$

where:

$$G = i \cot(\xi) \cos(k_m d_1)$$

where the coefficient of the Δv term is the sum of the impressed current and the current induced on the line by the slot antennas. For a single basis function Δv is a scalar, and if multiple basis functions are used Δv becomes a column vector consisting of the inner product of the fields in the slots due the feed line currents and basis functions of the magnetic currents in the slot. Now we can use the reciprocity relations given by equations 2.21 and 2.23 to eliminate A from this equation and obtain a direct solution of the magnetic currents in the slots.

Normally, this type of equation involves solving a set of linear equations, but if only one basis function per slot is used, the expression for the unknown coefficient, a, becomes (only one unknown for two slots because the currents are assumed to be the same amplitude):

$$a = \frac{G\,\Delta v}{Y_{11} + Y_{12} + 2H\,\Delta v^2} \qquad (2.32)$$

where:

$$H = \frac{i}{4\sin(\xi)} \left(\cos(k_m d_1)\cos(k_m d_2) + \cos(k_m d_1)^2 \right)$$

where Y_{11} is the admittance matrix element calculated by integrating the magnetic field due to the currents in slot 1 weighted by the basis function in slot 1, and Y_{12} is the matrix element obtained by integrating the field due the currents in slot 2 also weighted by the basis function in slot 1. If more than two basis functions are used, then the matrix form of the equation to be solved is:

$$G\,[\Delta v] = ([\,Y\,] + H\,[\Delta v]\,[\Delta v]^{T})\,[a] \tag{2.33}$$

where the superscript T denotes the transpose of the column vector $[\Delta v]$, and the column vector [a] contains the unknown weighting coefficients of the basis functions.

Although the analysis for the impedance calculations is much more involved, the approach of using transmission line models in the Green's functions for each plane wave component to simplify the expressions and aid in the interpretations of the behavior of the functions has been preserved. The desirable approach is to use the radiation models to optimize the dielectric structures and the type of antennas to use, then use the impedance calculations to determine the details of the feed network to obtain the best match to the detector.