A New, Accurate Quasi-Static Model for Conductor Loss in Coplanar Waveguide

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Conductor loss calculation techniques

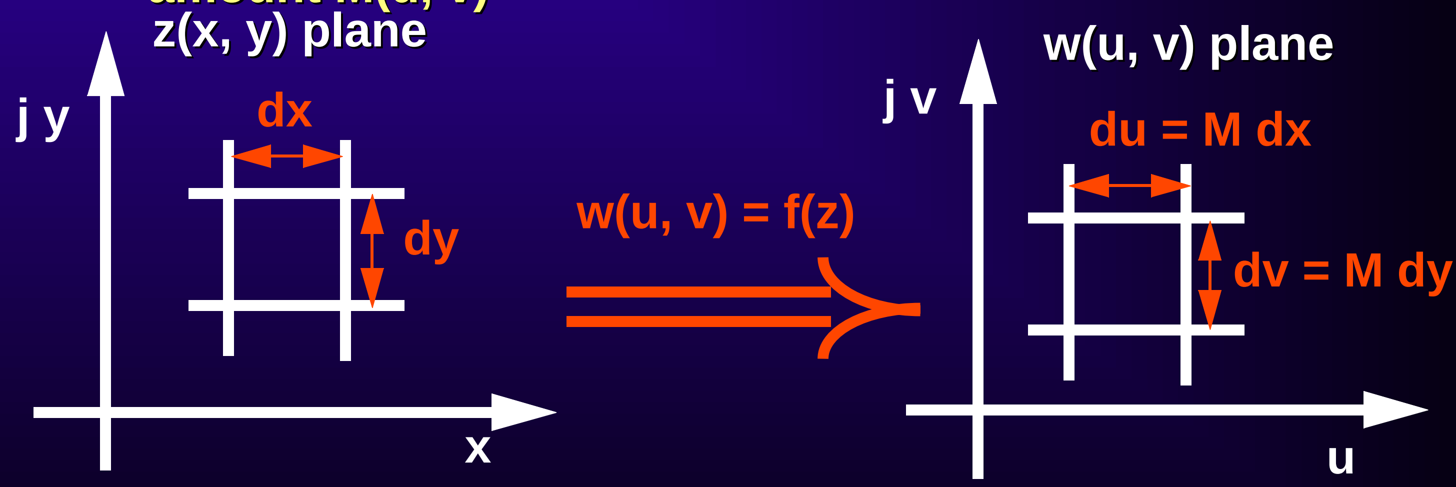
- Simple dc+skin depth resistance model
- Incremental inductance rule
  - Numerical evaluation (Wheeler)
  - Analytical evaluation (Gupta)
- Conformal mapping (Collin)
- Closed form expression (e.g., W. Heinrich, Trans. MTT, Jan.'93)
- Fullwave calculations

New Quasi-Static Technique

- Conformal mapping used to find frequency dependent series resistance and inductance, in addition to normal capacitance per unit length
- Requires the use of a "scaled" conductivity in mapped domain
  - surface impedance can be calculated using scaled conductivity
  - surface impedance can be found in real space, "scaled" in mapped domain

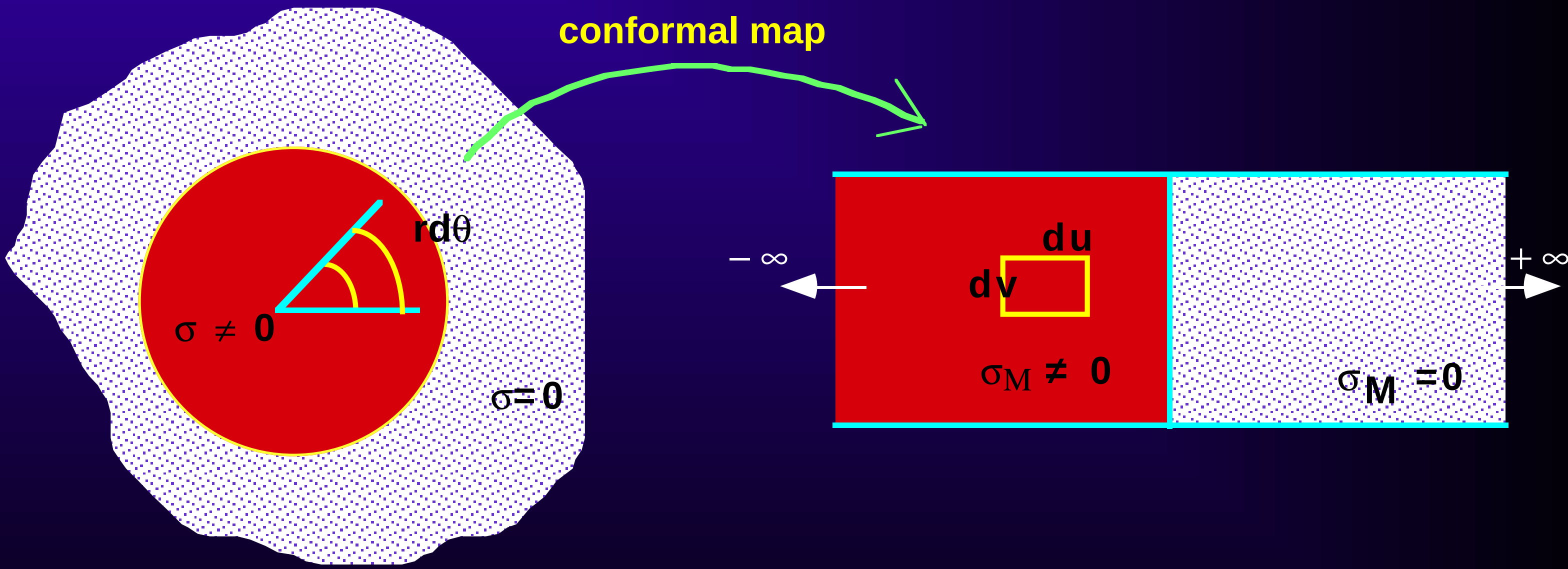
Length Scaling Under the Influence of a Conformal Map

- Conformal maps "preserve" local shape
  - differential lengths in w(u, v) plane are "scaled" by an amount M(u, v)



$$M(u,v) = \left|\frac{df}{dz}\right|_{u,v}$$

Example of scaling for a simple cylindrical wire



- Shunt capacitance depends on the ratio of side lengths :

$$\partial C = \frac{r\,\partial\theta}{\partial r} = \frac{(\partial v/M)}{(\partial u/M)} = \frac{\partial v}{\partial u}$$

  - scale factor cancels!
- Series resistance depends on the area :

$$\partial R = \frac{1}{\sigma}\,\frac{1}{r\,\partial r\,\partial\theta} = \frac{1}{\sigma}\,\frac{1}{(\partial u/M)\,(\partial v/M)} = \left(\frac{M^2}{\sigma}\right)\frac{1}{\partial u \partial v}$$

$$\therefore \sigma_M(\text{scaled}) = \frac{\sigma}{M^2}$$

  - scale factor enters as M^2!

Conductor Surface Impedance of a Circular Cylindrical Wire

- Can solve Helmholtz equation exactly

$$Z_s = -\frac{T\, J_o(Tr_o)}{2\pi r_o\ \sigma\ J_o'(Tr_o)} \qquad \text{where,}\ \ T^2 = -j\omega\mu\sigma$$

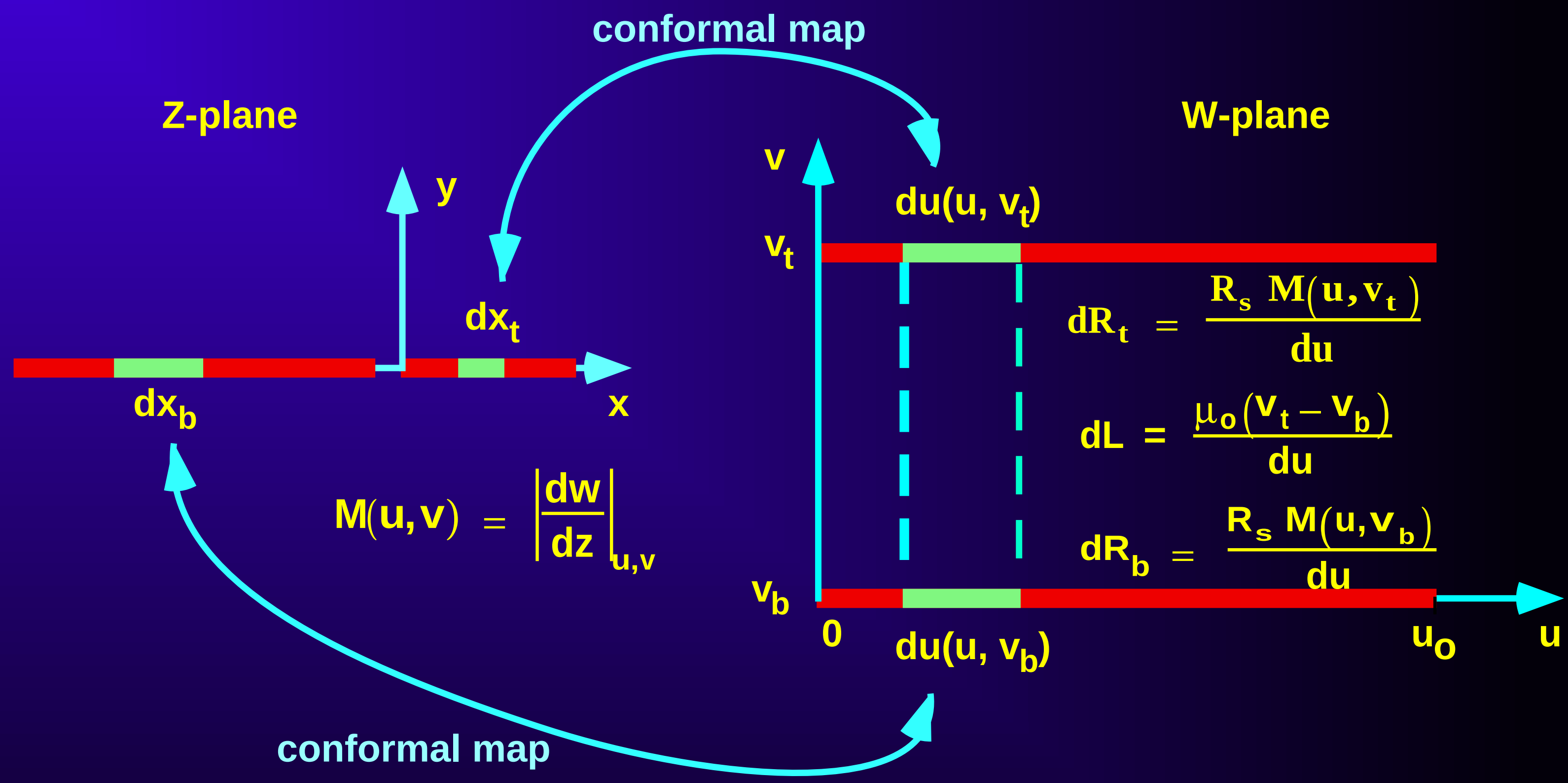
- In mapped domain requires solution for conducting rectangular slab with non-uniform conductivity
  - can be solved using non-uniform transmission line equations
- For both real & mapped planes, analytic expressions for Z_{surf} are identical

ARPA
Application Specific Electronic Modules Program

Conformal mapping & "scaling" in Co-planar waveguide

- Mapping is performed by evaluating elliptic integral of first kind
- Surface impedance is "scaled" in the mapped domain to include effect of current crowding
- Surface resistance is approximated as

$$R_s(\omega) = \mathbf{Re}\left[\frac{\sqrt{\frac{\omega\mu_o}{2\sigma}}\,(1+j)}{2\,\tanh\left\{\sqrt{\frac{\omega\mu_o\sigma}{2}}\left(\frac{t}{2}\right)(1+j)\right\}}\right]$$



- Diagram illustrating use of a conformal map to find the series impedance of a transmission line including the effect of finite resistance

- Series impedance Z due to differential width du is the series combination of mapped surface resistance and parallel plate inductance :

$$\partial Z = \frac{R_S\left\{M(u, v_t) + M(u, v_b)\right\}}{du} + \frac{j\omega\mu_o\left|v_t - v_b\right|}{du}$$

- Therefore, total series impedance per unit length is

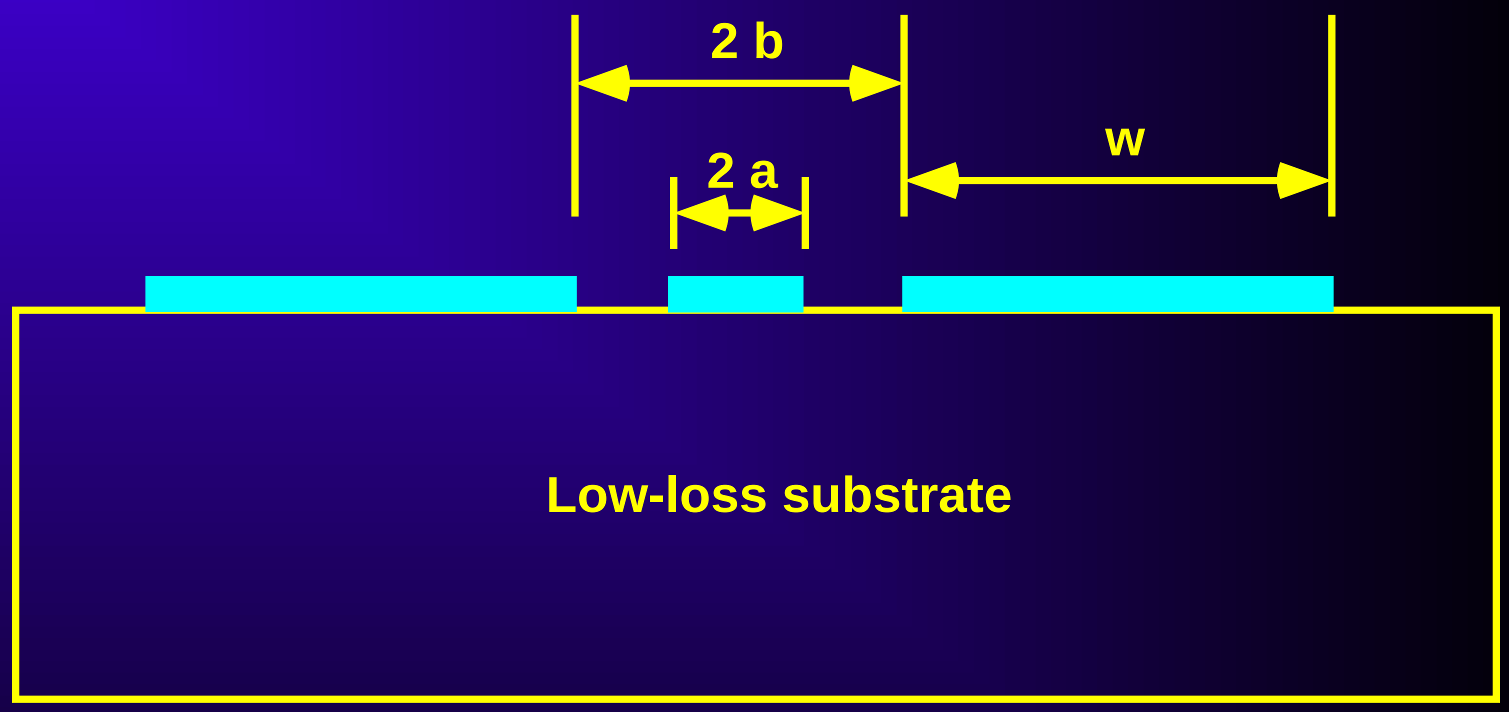
$$Z(\omega) = \left[\int_0^{u_o} \frac{du}{j\omega\mu_o\left|v_t - v_b\right| + R_s\left\{M(u,v_t) + M(u,v_b)\right\}}\right]^{-1}$$

- At low frequency total series impedance reduces to

$$Z = R_{dc} + j\omega\mu_o \left|v_t - v_b\right| \left(\frac{R_{dc}}{R_s}\right)^2 \int_0^{u_o} \frac{du}{\left[M(u,v_t) + M(u,v_b)\right]^2}$$

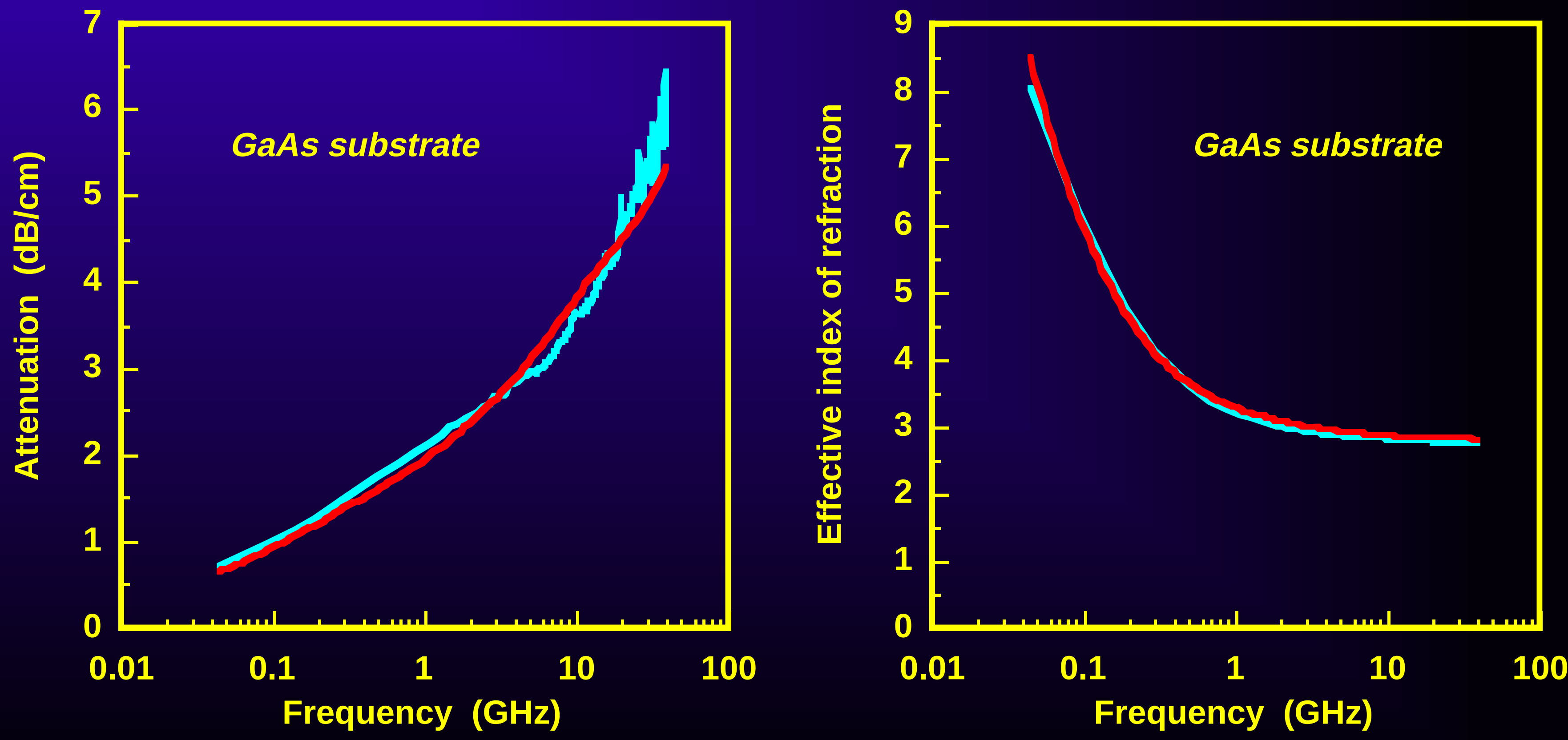
- At high frequency

$$Z = \frac{R_s}{u_o^{\,2}} \left\{ \int_o^{u_o} du \left[M(u,v_t) + M(u,v_b)\right] \right\} + j\omega L_{ext}$$

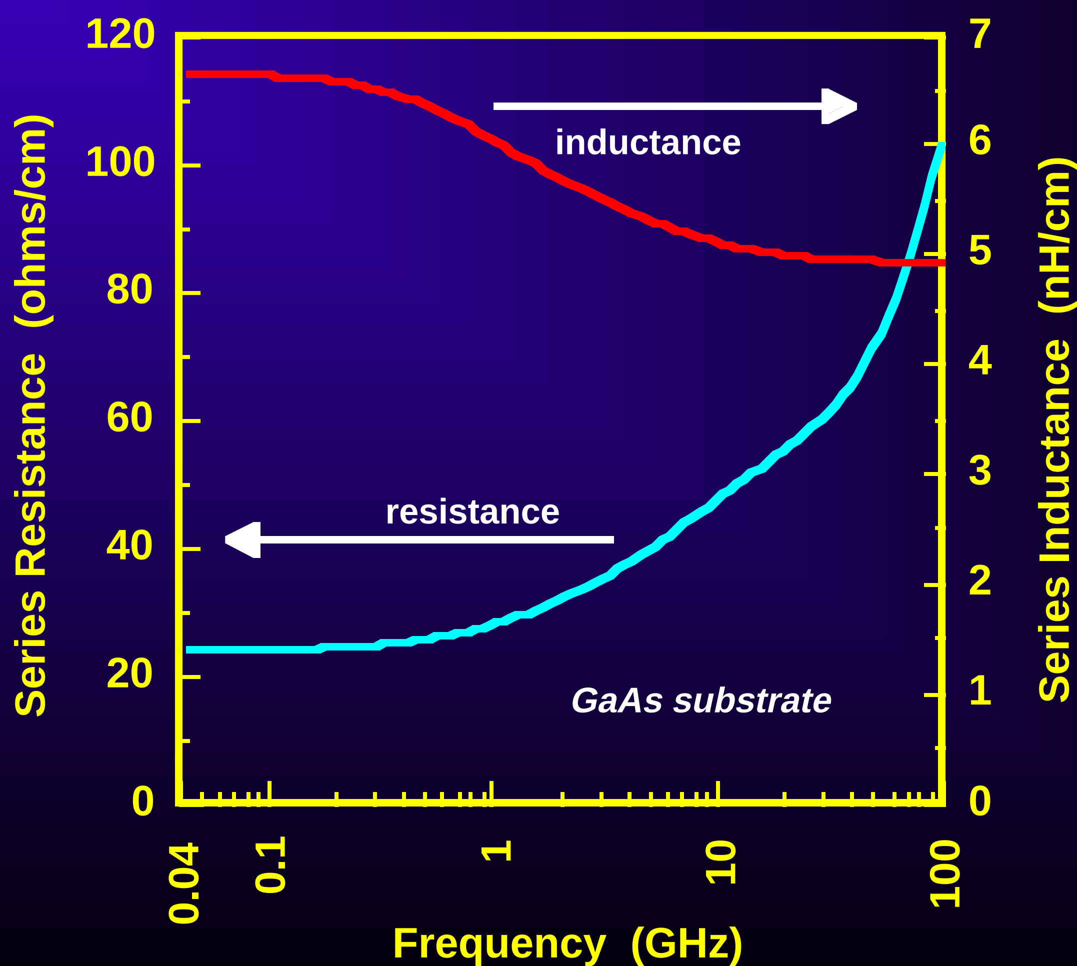


Cross-sectional view of CPW. Dimensions used for SI GaAs and Pyrex samples are a = 5 µm, b = 12 µm, w = 500 µm

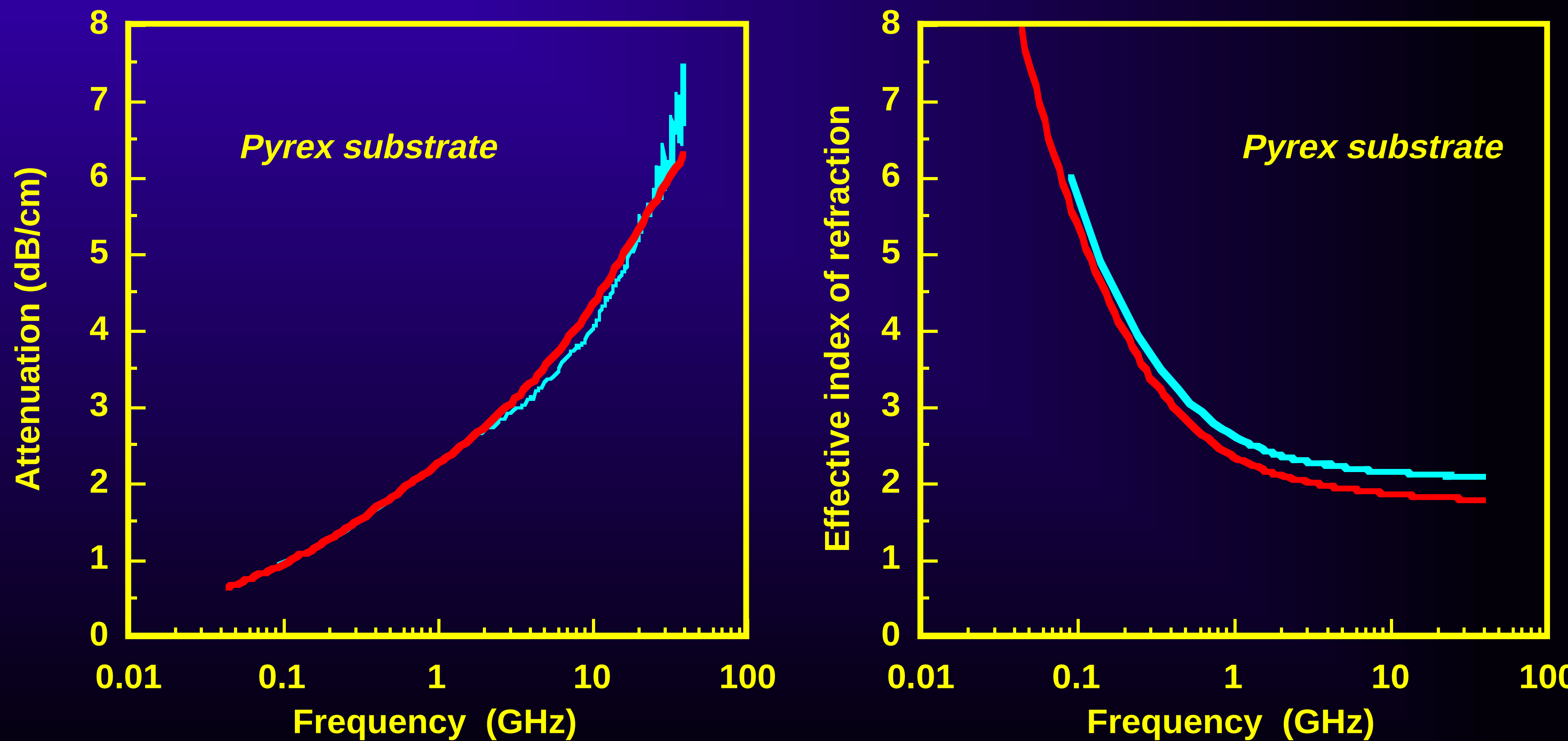
Comparison with our model: SI GaAs substrate



Equivalent series resistance and inductance: GaAs substrate



Comparison with our model: Pyrex substrate



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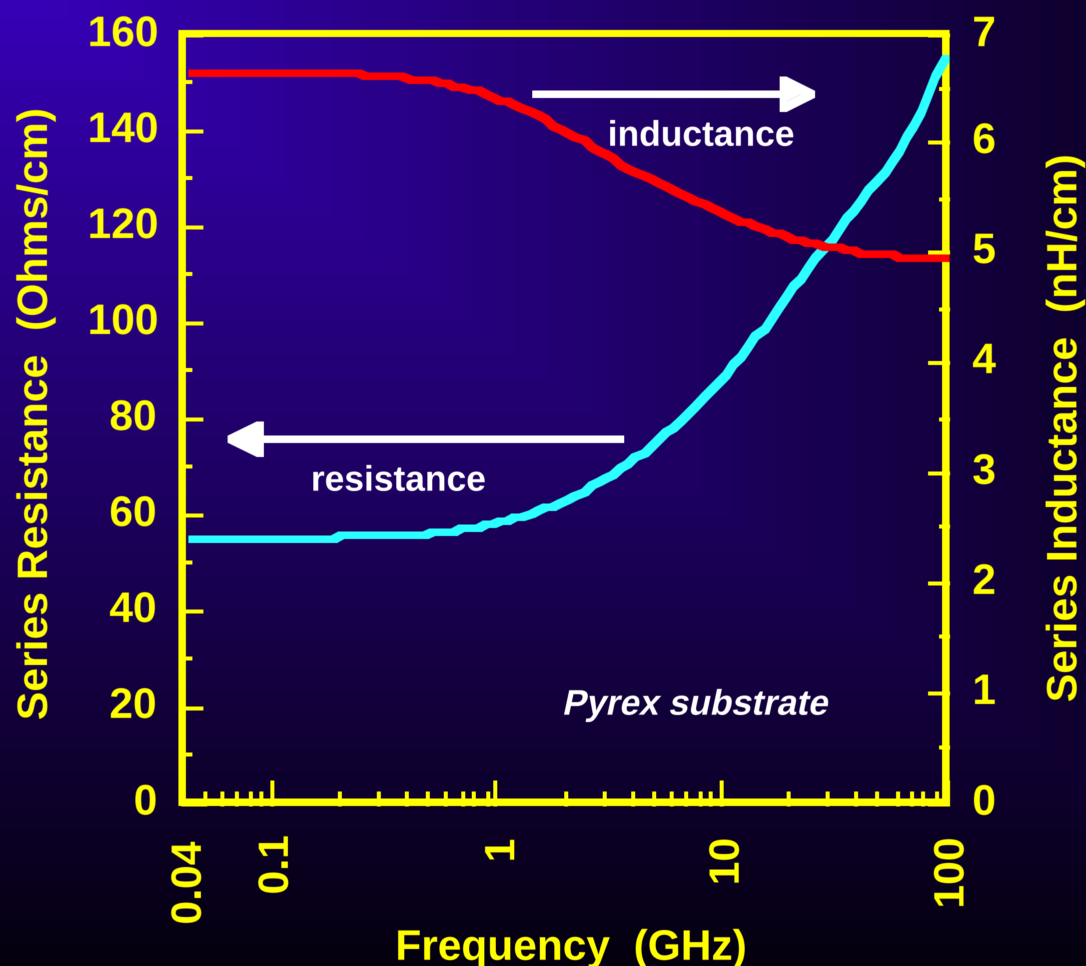
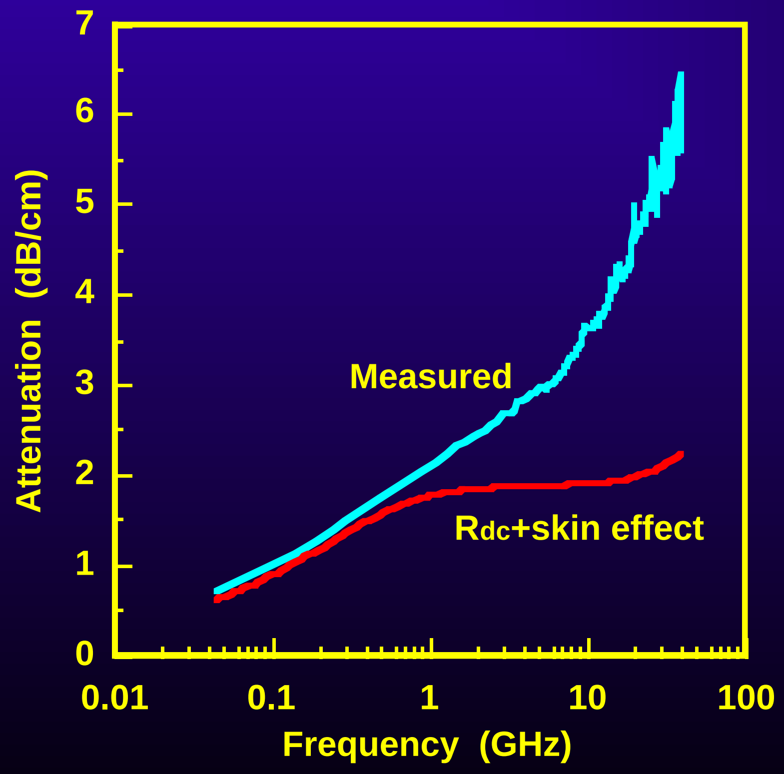
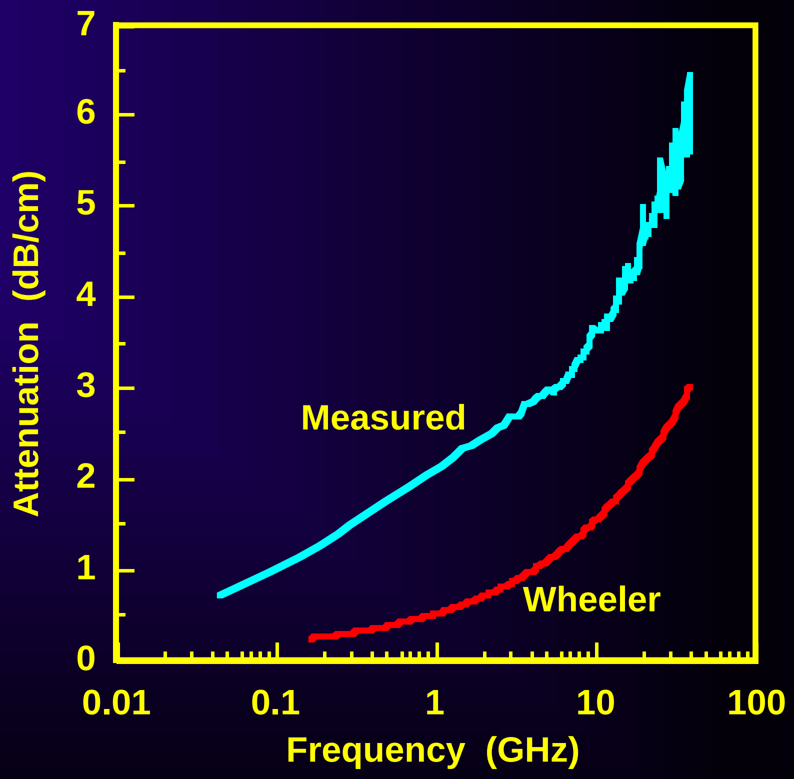


Comparison with other techniques (GaAs substrate)





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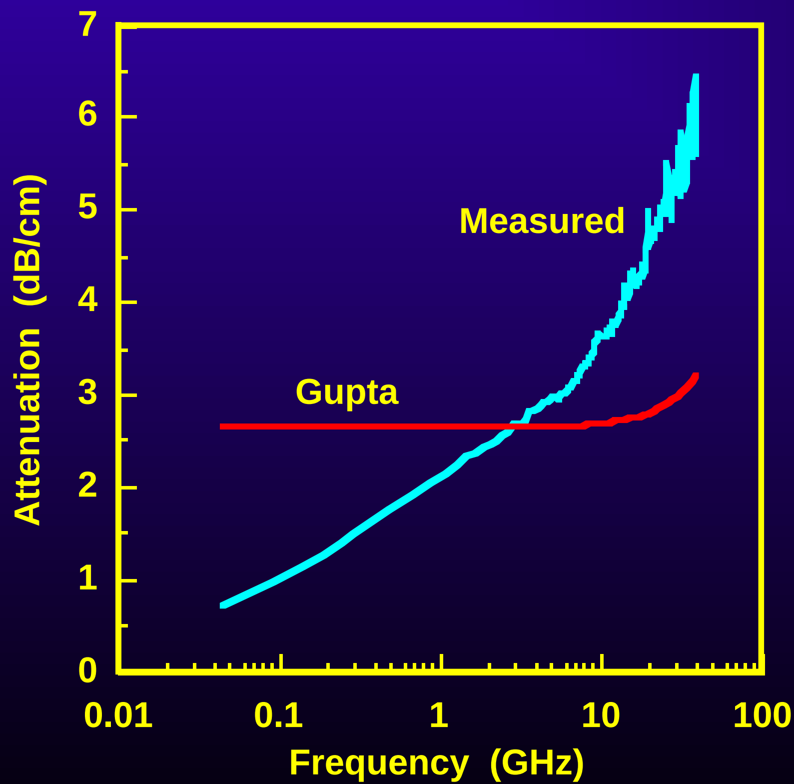
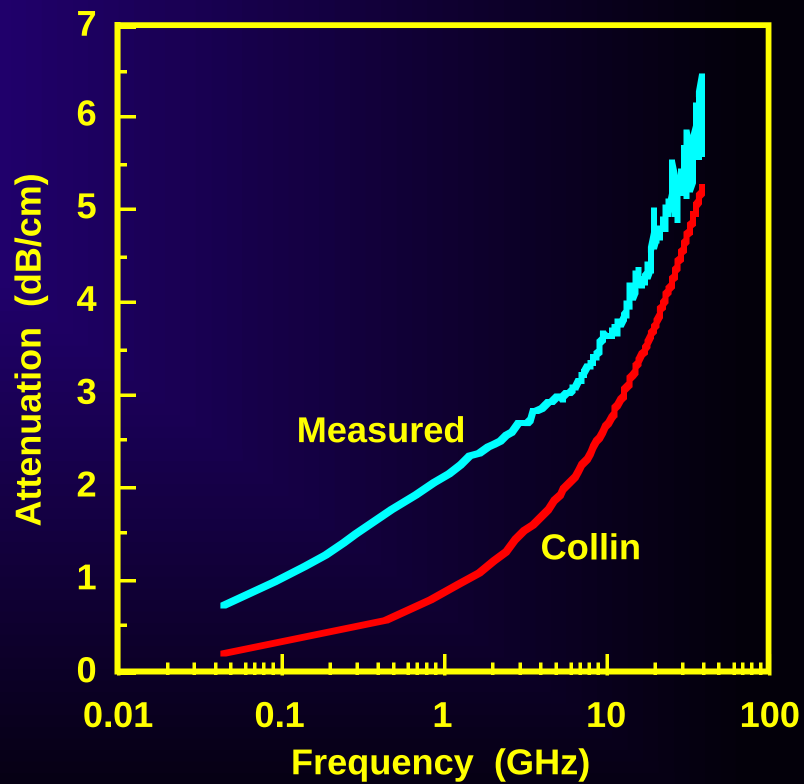
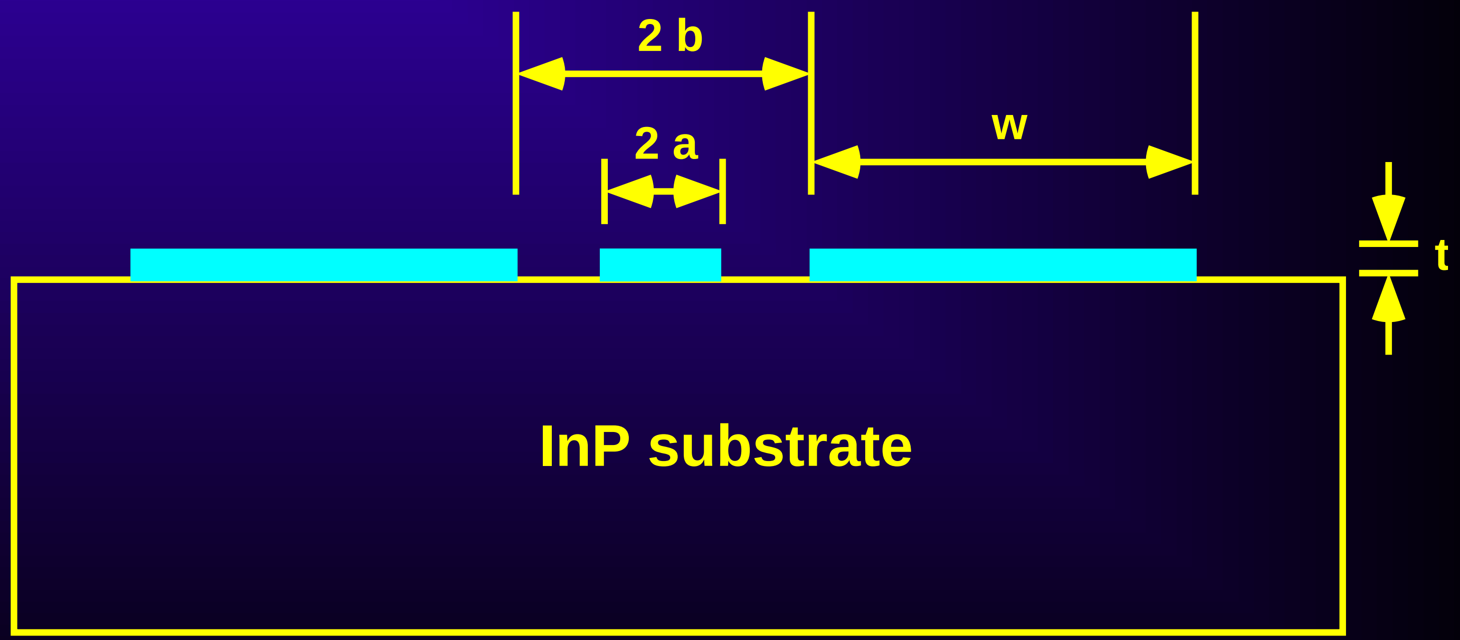




Comparison with Haydl's experimental data



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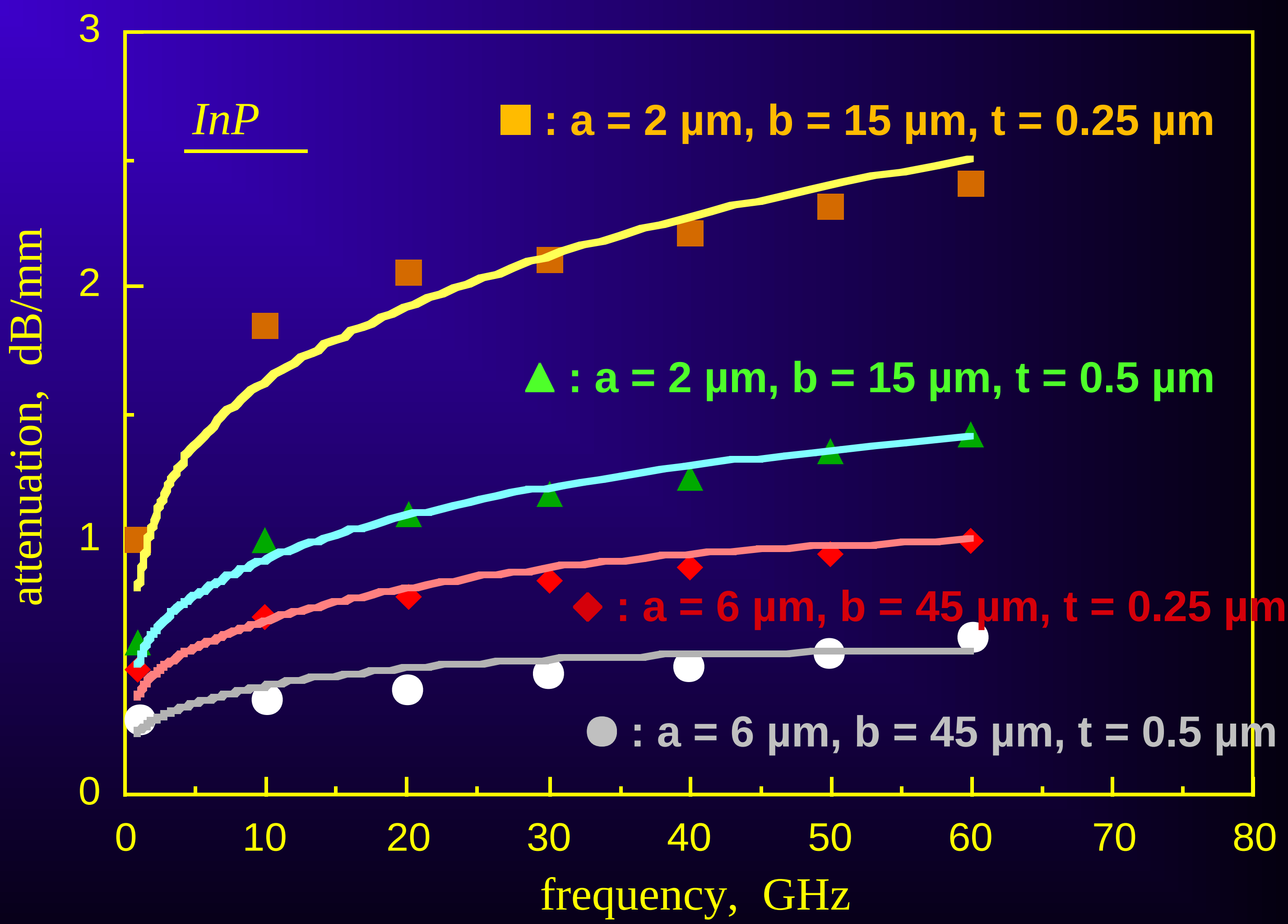
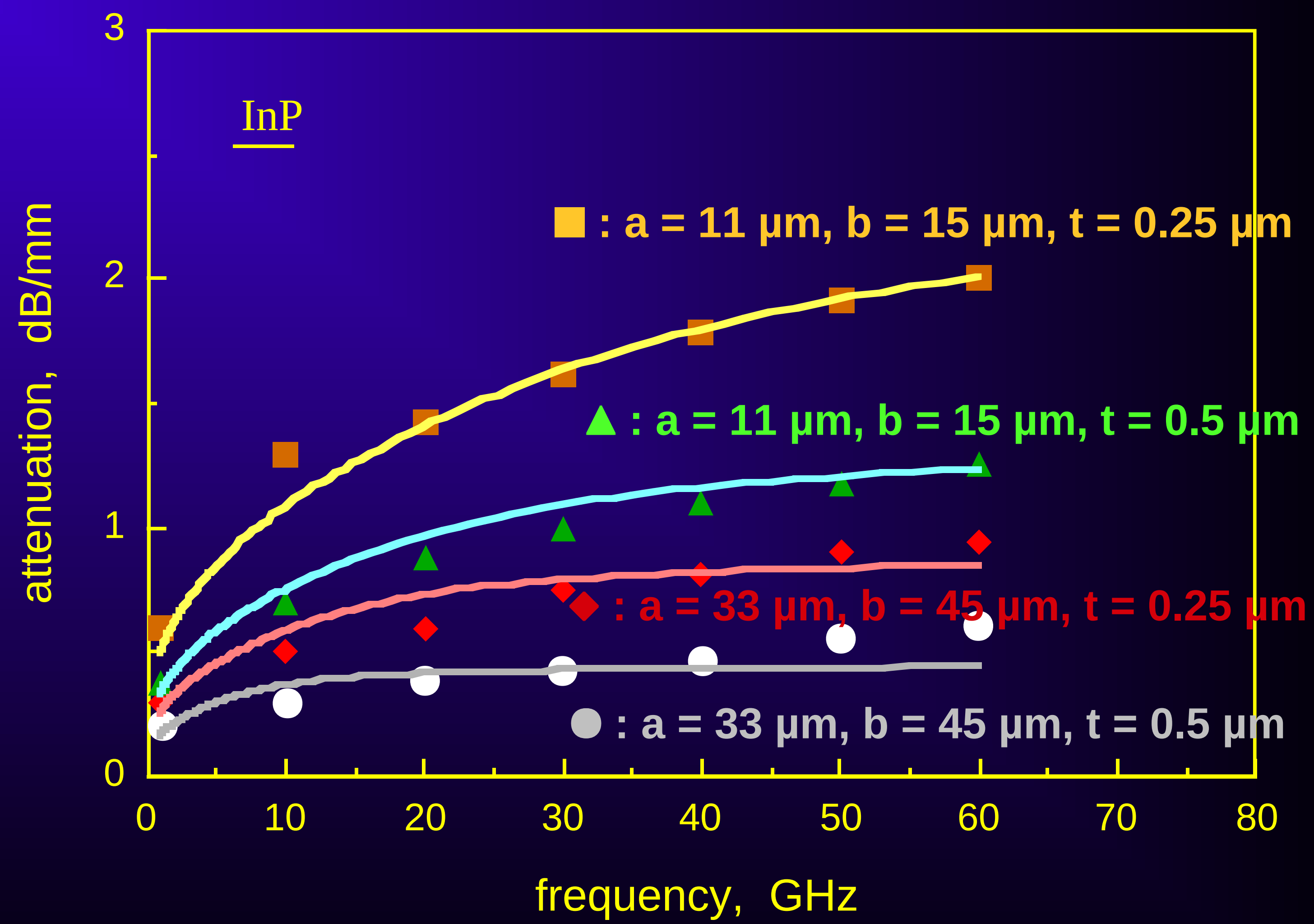


Application Specific Electronic Modules Program


InP
■ : a = 11 µm, b = 15 µm, t = 0.25 µm
▲ : a = 11 µm, b = 15 µm, t = 0.5 µm
◆ : a = 33 µm, b = 45 µm, t = 0.25 µm
● : a = 33 µm, b = 45 µm, t = 0.5 µm
attenuation, dB/mm
frequency, GHz
0
1
2
3
0
10
20
30
40
50
60
70
80

Conclusions

- Conformal mapping can be used to accurately predict conductor loss in quasi-TEM transmission lines
  - can be applied to coplanar strips, microstrip, strip-line
- Numerically efficient
- Closed form
- Easy to implement in CAD software
- Useful for interconnect loss calculation or timing analysis in Multi Chip Modules (MCMs)
  - generates equivalent circuit model that is very efficient for time domain simulation