Time Domain Multiconductor Transmission Line Analysis Using Effective Internal Impedance

Sangwoo Kim and Dean P. Neikirk

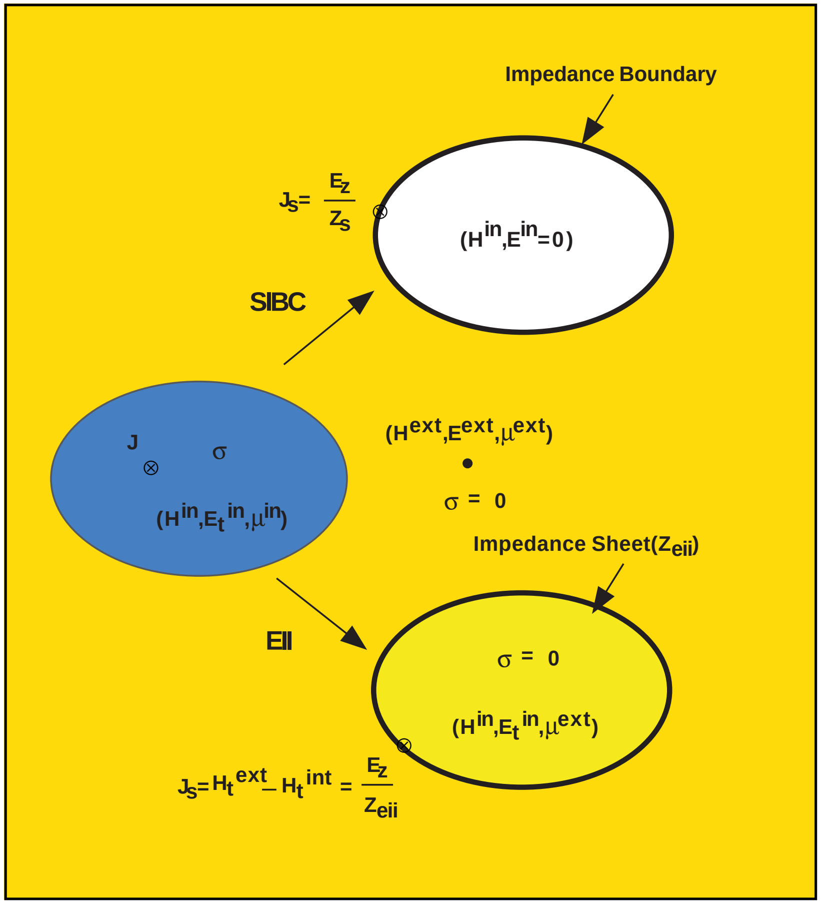
Department of Electrical and Computer Engineering

The University of Texas at Austin

WWW: http://weewave.mer.utexas.edu/

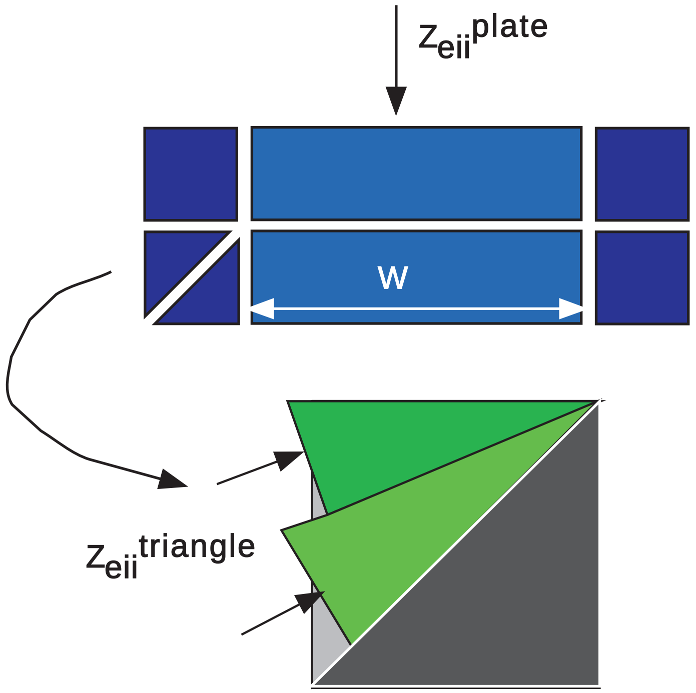
e-mail: neikirk@mail.utexas.edu

Frequency domain approach: using effective internal impedance(EII)



- not the same as standard surface impedance boundary condition (SIBC)
- inside of conducting material is replaced with external medium
- sheet current: produce original external field and non-zero inside field
- sheet impedance
  - -"Effective Internal Impedance"
  - - hard to calculate exactly
  - - can be approximated

Approximation of EII using transmission line model



- no unique approximation
- decompose bar into plate and triangular sections: effectively capture current crowding near the edge
- Zeii of each geometry determined by calculating input impedance

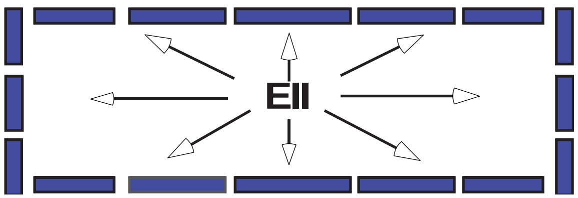
$$Z_{eii}^{plate} = \frac{(1+j)/(\sigma\delta)}{\tanh[(1+j)\cdot t/(2\delta)]}\cdot\frac{1}{W}$$

$$Z_{eii}^{triangle} = \frac{j(1+j)}{\sigma\delta}\,\frac{J_0\left(j(1+j)h/\delta\right)}{J_1\left(j(1+j)h/\delta\right)}\,\frac{1}{w}$$

Restrictions:

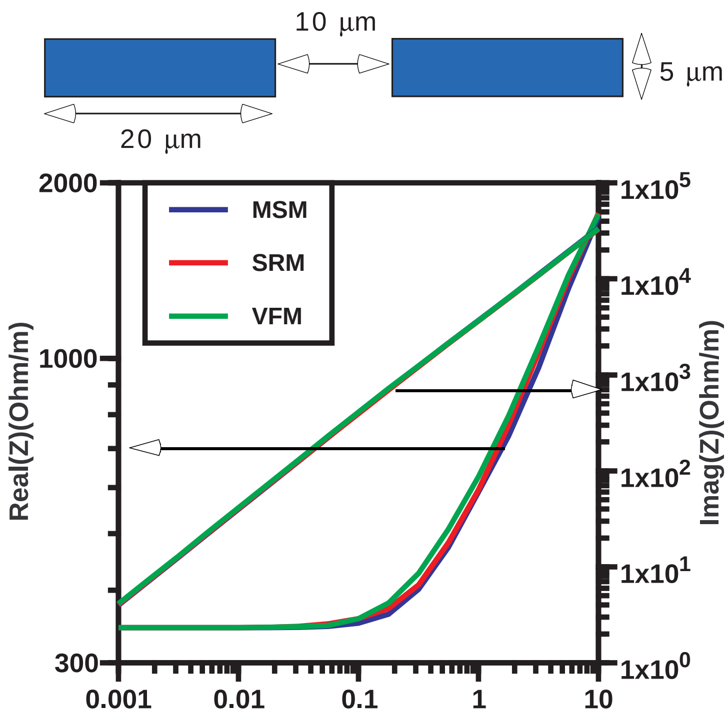
$$\Re e(Z_{eii})_{f\to 0} = \frac{1}{\sigma A} \qquad (Z_{eii})_{f\to\infty} = \frac{1}{\sigma\delta w}(1+j)$$

Surface ribbon method (SRM) in frequency domain



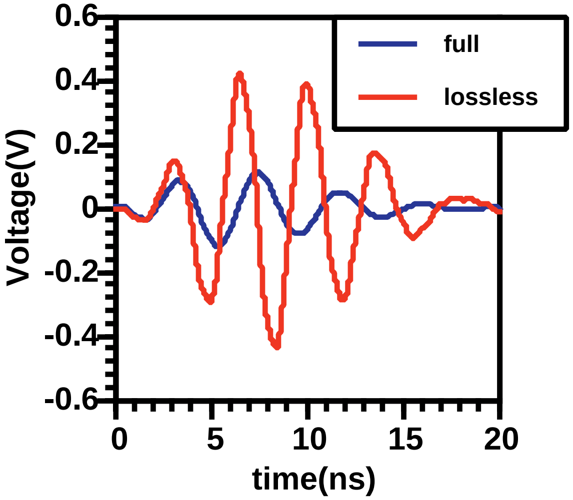
$$[Z_{eii}][I] + s[L][I] = -\frac{\partial}{\partial z}[V]$$

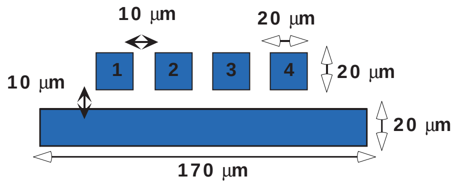
- EII assigned to each 'ribbon'
- mutuals between ribbons: capture external behaviors
- efficiency of SRM:
  - matrix size scales with conductor perimeter
  - minimum segmentation method

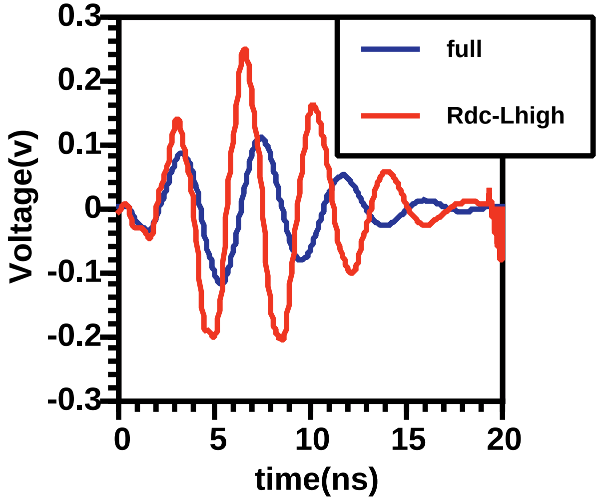


Unknowns: MSM-8, SRM-52, VFM-420

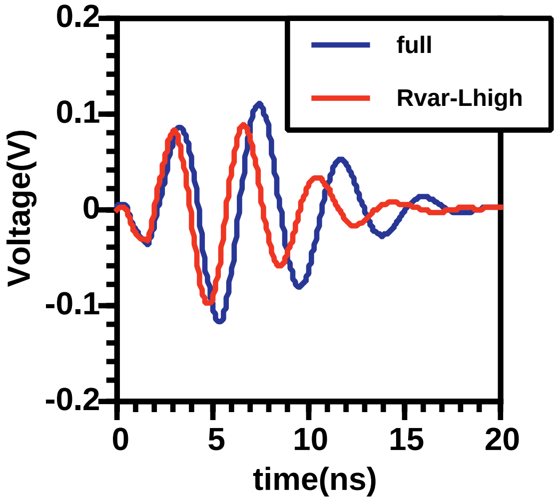
Effects of the frequency dependencies on time domain waveform



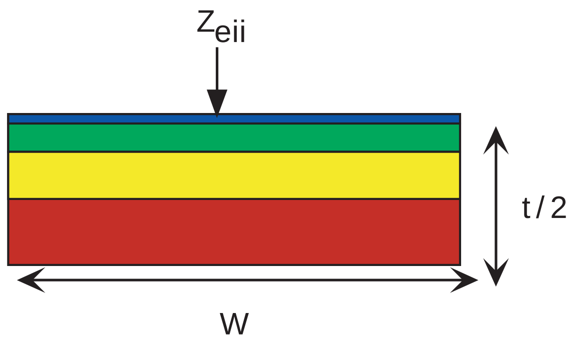


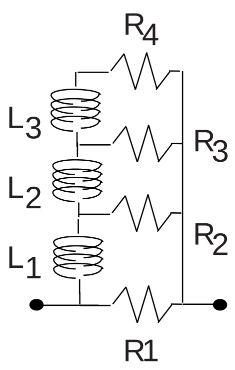


- line 1 excited with 0.1ns rise and fall time
- measured far end of line 4 (length=0.1 m)
- R_S= 5 Ohm, C_L= 10 pF



Equivalent circuit modeling for EII: transforming frequency domain EII into time domain





$$Z_{eii}(s) = R_1 \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

- each 'isolated conductor' cross-sections divided into 4 parts
- each part represented with 1 resistor and 1 inductor
- rules to determine values of circuit elements

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = RR \qquad \frac{L_1}{L_2} = \frac{L_2}{L_3} = LL$$

- additional constraints: correct DC resistance and inductance

- RR and LL are empirically determined constants unique to the geometry of the conductor

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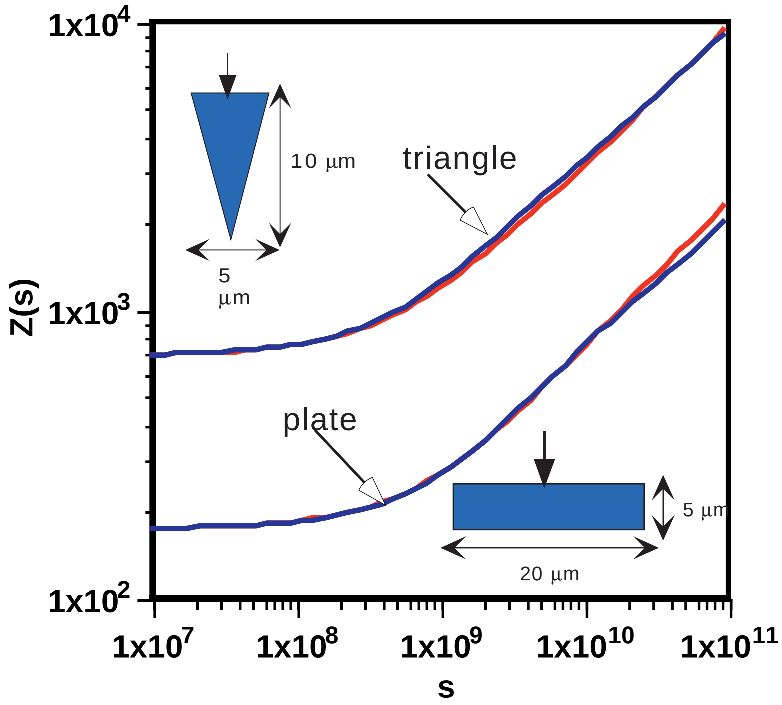
Time domain conversion using equivalent circuit model

- equivalent circuit model
  - can be easily constructed
  - rational function in s-domain, exponential function in time domain
  - problem size can be reduced using Pade approximation: dominant pole reduction
  - time domain convolution problem can be solved using recursive properties



blue: numerical result

red: circuit model

Derivation of time domain equation

- frequency domain (s-domain) equation

$$\left[\frac{Z_{eii}}{s}\right]s[I]+[L]s[I]=-\frac{\partial}{\partial z}[V]$$

- transformation into time domain

$$L^{-1}\left(\left[\frac{Z_{eii}}{s}\right]=\left[R_1\frac{b_3s^3+b_2s^2+b_1s+b_0}{s\left(a_3s^3+a_2s^2+a_1s+a_0\right)}\right]\right)$$
$$=\left[R_1\sum K_ie^{P_it}\right]=[\zeta(t)]$$

$$[\zeta(t)]*\frac{\partial}{\partial t}[I]+[L]\frac{\partial}{\partial t}[I]=-\frac{\partial}{\partial z}[V]$$

- time domain convolution

$$\mathrm{Y}(n\Delta t)=\mathrm{X}(n\Delta t)*Ke^{p(n\Delta t)}$$
$$=K\Delta t\cdot\mathrm{X}(n\Delta t)+e^{p\Delta t}\cdot\mathrm{Y}((n-1)\Delta t)$$

- application of recursive equation

$$[K]\frac{\partial}{\partial t}[I]+[L]\frac{\partial}{\partial t}[I]+[V_{ds}]=-\frac{\partial}{\partial z}[V]$$

$$\boxed{\begin{aligned}&[L']\frac{\partial}{\partial t}[I]+\underline{[V_{ds}]}=-\frac{\partial}{\partial z}[V]\\&[C]\frac{\partial}{\partial t}[V]+[G]\cdot[V]=-\frac{\partial}{\partial z}[I]\end{aligned}}$$

- lossless like equation with additional voltage source
- voltage source depends on poles, residues, time step, and values from previous time step
- different simulators can be used to solve equations

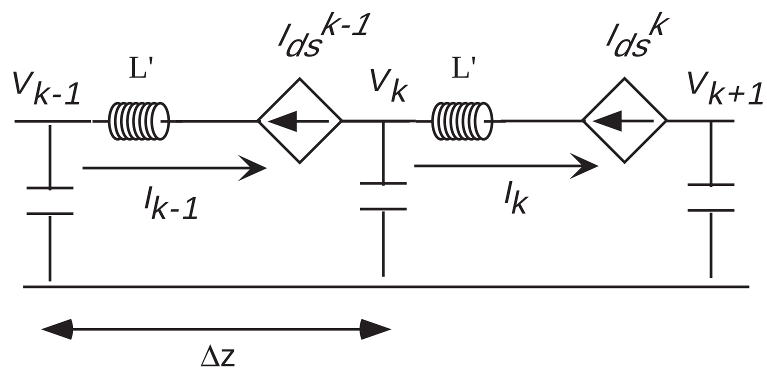
Example I: time stepping solution(FDTD)

- **make finite difference approximation to the partial derivatives: extra current source compared to lossless case**
- **each voltage and adjacent current solution point separated by Δz/2**
- **Δt has to be kept small to satisfy stability condition: may not be appropriate for electrically long lines**

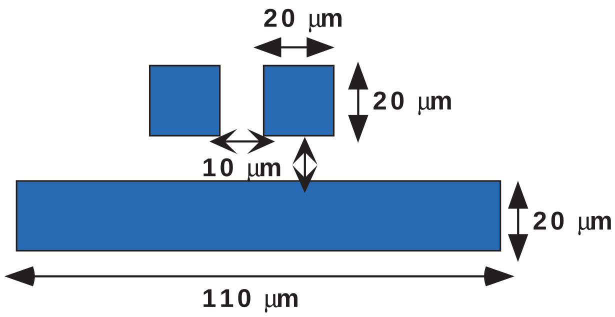
$$[\boldsymbol{I}]_k^{n+\frac{2}{3}} = [\boldsymbol{I}]_k^{n+\frac{1}{2}} - \left[\frac{\boldsymbol{L}'}{\Delta \boldsymbol{t}}\right]^{-1}\left(\frac{[\boldsymbol{V}]_{k+1}^{n+1} - [\boldsymbol{V}]_k^{n+1}}{\Delta \boldsymbol{z}} + [\boldsymbol{V}_{ds}]_k^{n+1}\right)$$

$$= [\boldsymbol{I}]_k^{n+\frac{1}{2}} - \left[\frac{\boldsymbol{L}'}{\Delta \boldsymbol{t}}\right]^{-1}\left(\frac{[\boldsymbol{V}]_{k+1}^{n+1} - [\boldsymbol{V}]_k^{n+1}}{\Delta \boldsymbol{z}}\right) - \underline{[\boldsymbol{I}_{ds}]}_k^{n+1}$$

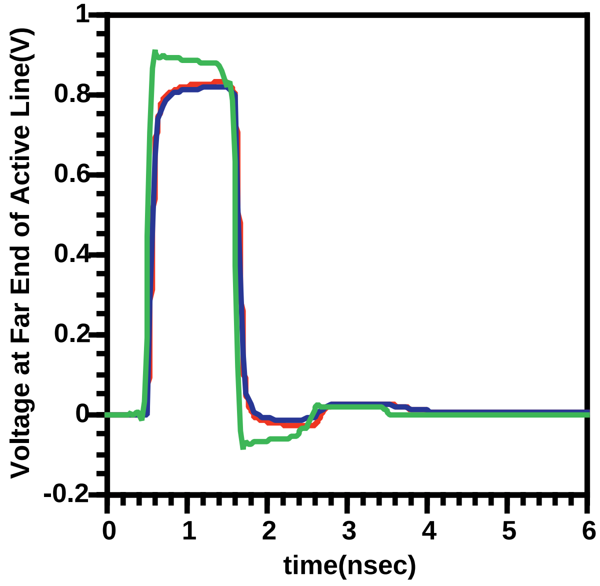
$$[\boldsymbol{V}]_k^{n+1} = [\boldsymbol{V}]_k^{n} - \left[\frac{\boldsymbol{C}}{\Delta \boldsymbol{t}}\right]^{-1}\left(\frac{[\boldsymbol{I}]_{k+1}^{n+\frac{1}{2}} - [\boldsymbol{I}]_k^{n+\frac{1}{2}}}{\Delta \boldsymbol{z}}\right)$$

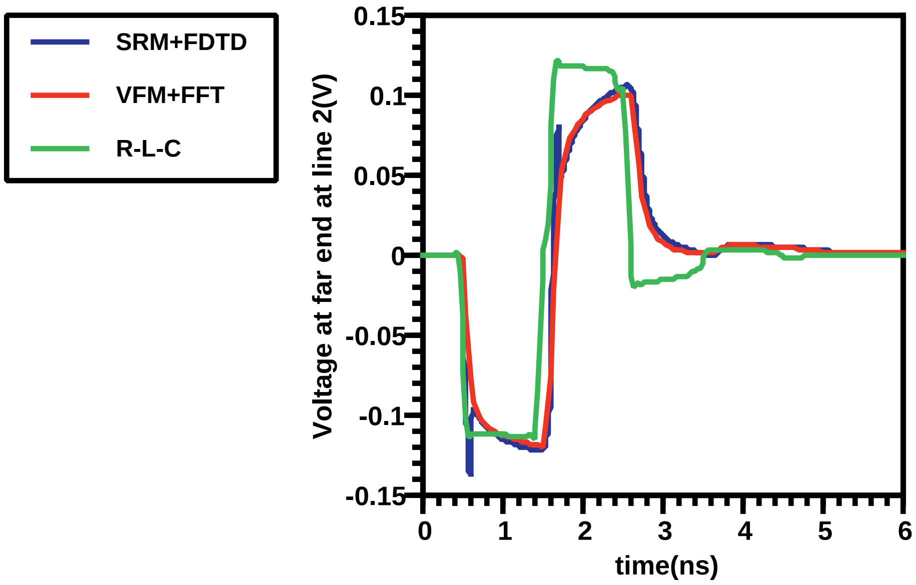


Example I: continued



- line 1 excited with 0.1 ns rise and fall time trapezoidal waveform
- line length= 0.1m, R_S= 5 Ohm, R_L= 50 Ohm





Using EII for time domain simulation

- frequency domain concept (SRM) can be easily applied to time domain using equivalent circuit for EII
- excellent agreement with FFT calculations
- significant decrease in run-times demonstrated
- computation time can be further reduced: run time can be comparable to simple R-L-C circuit analysis
  - dominant pole approximation
  - minimum segmentation
- can easily include dc-to-skin effect behavior directly in time domain models